A THEORETICAL STUDY OF THE COMPOSITION OF THE ALVEOLAR AIR AT ALTITUDE

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In our studies of high altitude physiology we have found an oxygen-carbon dioxide diagram a very great aid to accurate thinking. It has been useful in so many different problems that we consider it worth while to describe it in this paper so that it can be made available to others.

1. Breathing pure oxygen. In this diagram the alveolar tensions of carbon dioxide are plotted as ordinates against the alveolar tensions of oxygen as abscissae as in figure 1, which represents the situation when an aviator ascends to high altitudes breathing pure oxygen. At any altitude, breathing pure oxygen,

\[ p'CO_2 + p'O_2 = p'O_2 \]

where \( p'O_2 \) refers to the tension of oxygen in the inspired air (BTPS or body temperature, saturated, ambient pressure) and the other tensions refer to alveolar air (BTPS). This equation gives a family of parallel diagonal lines on the chart each of which represents a given altitude such that \( B - 47 = p'O_2 \). The intercepts of these diagonals on the \( X \) axis represent therefore values of \( B - 47 \). With the aid of Henderson's nomogram for the blood of A.V.B. or any other similar data it is possible to assign a given percentage saturation of the arterial blood to every point on the chart. Thus lines of equal arterial saturation can be drawn. To indicate the location of such a family of curves the 65, 75, 85 and 95 per cent saturation lines have been drawn. When the saturation becomes less than about 65 per cent the subject is almost certain to lose consciousness very soon and this region of the chart has therefore been labelled "anoxia."

To show the normal behaviour of an aviator in going to progressively higher altitudes the line marked "alveolar air" has been drawn to indicate the successive positions of the alveolar air composition as the altitude is increased from ground level to above 43,000 feet. The curve bends downward after passing the 95 per cent saturation line because of the hyperventilation caused by the stimulus of anoxia. Hyperventilation at 43,000 feet moves the alveolar point

\[ p'CO_2 + p'O_2 = p'O_2 \]

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down on the corresponding diagonal so that the $pCO_2$ is decreased about 10 mm. Hg and the $pO_2$ is increased a corresponding amount. If the $pCO_2$ is lowered too far by hyperventilation the symptoms of acapnia develop. A line marking this threshold has been drawn on the diagram at 20 mm. Hg although the level varies considerably with different individuals and different durations of exposure.

So long as the rate of CO$_2$ output remains constant alveolar $pCO_2$ will vary inversely with alveolar ventilation. On this assumption a scale of ventilation rates has been indicated at the right of the diagram. The values were calculated according to the equation

$$V'_a = \frac{272}{pCO_2}$$

where 272 is a constant (see equation 17 and discussion), $V'_a$ is the alveolar ventilation (BTPS) in liters per minute and

$$V'_a = \text{total ventilation} (V_t) - \text{(dead space} \times \text{frequency})$$

Actually of course the CO$_2$ output does not remain constant at altitudes where hyperventilation occurs because extra CO$_2$ is blown off. Temporarily this will invalidate the equation given above until the R.Q. returns to normal. A rigid derivation of a ventilation equation to take account of this situation will be given below.
The chart in figure 1 represents the simplest condition. When air instead of oxygen is breathed the diagram is much more complicated but it is for the understanding of these same complications that the diagram is particularly useful.

2. The alveolar air equation. To understand the air-breathing diagram it is necessary to develop the alveolar air equation.

Let $V =$ a volume of moist inspired air in milliliters
$y =$ milliliters of moist CO$_2$ added to $V$ in the lungs
$x =$ milliliters of moist O$_2$ removed from $V$ in the lungs
$C =$ CO$_2$ and $O =$ O$_2$
$f =$ the fraction of moist gas in alveolar air (by volumes)
$pC$ and $pO =$ partial pressures of $C$ or $O$ in alveolar air in mm. Hg
$f'$ and $p'$ refer to moist inspired air
$Q =$ respiratory quotient as observed $= \frac{y}{x}$

$V + y - x =$ the new volume after transition from tracheal to alveolar air.
$y = fC (V + y - x) - f'CV$
$x = f'OV - fO (V + y - x)$

Substituting $Qx$ for $y$ and $\frac{y}{Q}$ for $x$

$$x = \frac{f'OV - fOV}{1 + fOQ - fO}$$ (3)

$$y = \frac{fCV - f'CV}{1 - fC + \frac{fC}{Q}}$$ (4)

Substituting these values of $x$ and $y$ in the equation $\frac{y}{x} = Q$ and solving for $fO$
gives

$$fO = \frac{f'OQ + fCf'O(1 - Q) + f'C - fC}{f'C(1 - Q) + Q}$$ (5)

Multiplying both sides by $(B - 47)$ to change $f$ to $p$ we have

$$pO = \frac{p'OQ + pCf'O(1 - Q) + p'C - pC}{f'C(1 - Q) + Q}$$ (6)

This is the complete alveolar air equation but if CO$_2$-free gas is inhaled and $f'C = 0$ it can be simplified to$^2$

$$pO = p'O + \frac{pCf'O(1 - Q)}{Q} - \frac{pC}{Q}$$ (7)

If pure oxygen instead of air is inhaled and $f'O = 1$; or if $Q = 1$, whatever the value of $f'O$ then

$$pO = p'O - pC$$ (8)

$^2$ This same equation has been given by Boothby in the Handbook of Respiratory Data, (Washington, D. C., 1944) where prior credit is given to several other authors. (See Essay A.)
Since equation 8 is identical with equation 1 used for the altitude diagonals in figure 1 it is evident (1) that this chart is applicable to oxygen-breathing altitudes regardless of the RQ and (2) that if the R.Q. = 1.0 the chart is also applicable to the problems involved in the breathing of air.

3. Breathing air. Equation 7 represents the condition where CO₂-free air is inhaled which is practically true of ordinary air. The application of this equation to the oxygen-carbon dioxide diagram is illustrated in figure 2. The altitude of 10,000 feet is now represented by a family of diagonals all radiating from a pO₂ value of 100 mm. Hg, this being the oxygen tension in the moist inspired air. There is a different diagonal for each R.Q. value and at each altitude there is a similar set of diagonals. Four such sets of Q diagonals have been drawn in figure 2 for four different altitudes. The position of the alveolar air on any one of these diagonals will depend upon the rate of breathing and the rate of CO₂ output. The scale on the right indicates the alveolar ventilation for each pCO₂ level on the assumption that the CO₂ output is 315 cc. per minute (S.T.P.). The average alveolar air line is based upon analyses of large numbers of samples exhaled forcibly beginning at the end of a normal expiration. Data will be published elsewhere.

It is obvious that the diagonals of one altitude intersect some of the diagonals of a slightly higher or lower altitude. At one of these points of intersection therefore the subject might be at 20,000 feet with an R.Q. of about 1.4 due to

![Figure 2. Alveolar pCO₂ and pO₂ at different altitudes breathing air. Familiar of diagonals for 4 different altitudes are shown. Each line represents a certain R.Q. value as indicated on the graph and was calculated from equation 7. The scale of alveolar ventilation at the right is calculated by equation 17 on the arbitrary assumption that the rate of CO₂ output is 315 cc. per minute (S.T.P.). The average alveolar air line is based upon analyses of large numbers of samples exhaled forcibly beginning at the end of a normal expiration. Data will be published elsewhere.](http://ajplegacy.physiology.org/DownloadedFrom)
overventilation and a lowered $pCO_2$ or he might be at 15,000 feet with an R.Q. of about 0.8. This illustrates the difficulty of calculating what altitude is equivalent to a given low-oxygen nitrogen mixture. Strictly speaking it is impossible to calculate such equivalent altitudes without specifying the rate of breathing and the R.Q. (See also fig. 5.)

A full set of altitude diagonals for 20,000 feet breathing air is shown in figure 3. If equation 7 is solved for $Q$

$$Q = \frac{pC(1 - f'O)}{p'O - pO - f'OpC}$$  \hspace{1cm} (9)

Fig. 3. Alveolar air diagram for 20,000 feet breathing air. Each diagonal line starting at $pO_2 = 63$ represents a given R.Q. as indicated on the diagram and was calculated from equation 7. The nearly vertical parallel lines represent points of equal alveolar ventilation calculated by equation 15 on the arbitrary assumption that the rate of oxygen consumption, $X_0$, equals 315 cc per min. The average alveolar air line is the same as that in figure 2. The R.Q. = $\infty$ and $V_a = \infty$ lines are identical and have a reciprocal slope of 0.2093 = $f'O$. For points A, B and C see text p. 646.

Obviously $Q = \infty$ when the denominator = 0 or when $p'O - pO = f'OpC$. The negative (reciprocal) slope of the $Q = \infty$ diagonal is therefore $f'O$. The iso-ventilation lines which are also included in figure 3 will be discussed later.

It is instructive to inquire what happens to the R.Q. diagonals when the percentage of oxygen in the inspired air increases. For the sake of simplicity it may be assumed that the barometric pressure is so adjusted that at all percentages the tension of oxygen in the inspired air ($X$ intercept) remains constant. This assumption however is immaterial to the argument because slopes of the R.Q. diagonals are identical at all values of $p'O$ for the same value of $f'O$. 

From figure 4 it can be seen that when \( f'O \) (i.e., the fraction of \( O_2 \) in the inspired air) increases, the R.Q. diagonals come closer together until when \( f'O = 1.0 \) they all coincide with the R.Q. = 1.0 diagonal. Thus when pure \( O_2 \) is breathed the alveolar air must lie along a single diagonal regardless of the R.Q. On the graphs in figure 4 two special alveolar points are indicated by circles: (1) at \( pCO_2 = 40 \) and \( Q = 0.8 \) and (2) at \( pCO_2 = 25 \) and \( Q = 1.5 \). Point 1 is a normal and point 2 is one that could easily be reached temporarily by overventilation without danger of serious hypocapnia. It is significant that the gain in \( pO_2 \) in this process is 33 mm. Hg at 6 per cent, 31 mm. at 21 per cent, 24 mm. at 60 per cent, 20 mm. at 80 per cent \( O_2 \) and only 15 mm. at 100 per cent \( O_2 \). At air-breathing altitudes therefore the immediate benefit (in terms of \( pO_2 \) gain) derived from overventilation may be twice as great in terms of alveolar oxygen tension as at oxygen-breathing altitudes and this difference depends upon the spreading of the R.Q. diagonals.

4. Equivalent altitudes. The difficulty involved in calculating what mixture of \( O_2 \) and \( N_2 \) is equivalent to a given altitude is illustrated by figure 5. The solid lines represent R.Q. diagonals at 20,000 feet breathing air. The same inspired \( p'O_2 \) can be obtained by an 8.83 per cent \( O_2 \) + 91.17 per cent \( N_2 \) mixture.
at sea level but the R.Q. diagonals on such a mixture differ appreciably as indicated by the dotted lines. At equal rates of ventilation therefore the alveolar air would not be the same under the two conditions unless the R.Q. = 1.0. Furthermore at the same R.Q. (except when $Q = 1$) it would be impossible to breathe in such a way that the alveolar composition would be the same at ground level on low $O_2$ as at altitude on air. This is obvious because the R.Q. diagonals for the two conditions do not cross except at infinite ventilation rates (i.e., at $pC = 0$). This illustrates convincingly the possible fallacies involved in the common expedient of simulating high altitude conditions by low oxygen mixtures.

![Diagram](http://ajplegacy.physiology.org/)

Fig. 5. Difference between 20,000 feet breathing air (solid lines) and an 8.85 per cent oxygen mixture in nitrogen at ground level having the same inspired $pO_2$. The dotted and solid R.Q. diagonals diverge more and more as the R.Q. becomes greater or less than 1.0. This illustrates the problem of calculating equivalent altitudes. See text. Each line is calculated according to equation 7.

Similar difficulties are encountered if the condition of breathing air at 20,000 feet is simulated by inhaling pure $O_2$ at 100 mm. Hg where the inspired $pO_2$ is again the same. In this case the alveolar air must lie along the $Q = 1$ diagonal regardless of the actual value of $Q$, and the discrepancies between the two conditions are therefore still larger than in the previous comparison. In order to simulate 20,000 feet breathing air by pure $O_2$ at a higher altitude it is necessary to select an altitude for pure $O_2$ such that the $B - 47$ value is equal to $pC + pO$ for the alveolar air at 20,000 feet. If then the R.Q. and CO$_2$ output can be assumed to remain the same at both altitudes, the alveolar air will be the same provided the ventilation is so regulated that the alveolar $pCO_2$ is also the same.

5. The ventilation equation. The intercept of one of these R.Q. diagonals on the $x$ axis represents the oxygen tension of the moist inspired air. When the air enters the alveoli the oxygen tension diminishes and the carbon dioxide ten-
sion increases and the point on the chart which represents the composition of
the alveolar air moves up to the left along one of these diagonals. If the rate
of breathing is very fast it will not move very far up the diagonals because the
air will not remain long in contact with the alveolar walls. If the rate of breath-
ing is "normal" the \( pCO_2 \) will be "normal" and the alveolar air composition
will lie along the average alveolar air line shown in the chart (fig. 2 or 3). If
the rate of breathing is infinitely fast the alveolar air will have the same com-
position as the inspired air. It should be possible, therefore, to draw a series of
lines across the chart which represent equal rates of ventilation. The derivation
of equations for plotting these lines must now be considered. At air breathing
altitudes this is somewhat more complicated than it is for pure oxygen.
The problem is much simplified in the beginning if only alveolar ventilations are
concerned. The total ventilation can always be obtained by adding the dead
space ventilation which is equal to the product of dead space volume and the
frequency of breathing. Let \( Y \) and \( X \) = rates of CO\(_2\) output and \( O_2 \) intake
respectively in cubic centimeters per minute at BTPS.

\[
X_o = O_2 \text{ intake in cc. per minute at } 0^\circ C. \text{ dry, } 760 \text{ mm. Hg}
\]

\[
Y_o = \text{CO}_2 \text{ output in cc. per minute, at } 0^\circ C. \text{ dry, } 760 \text{ mm. Hg}
\]

\[
V_a = \text{Alveolar ventilation in cc/min at BTPS, } V_t = \text{total and } V_d = \text{dead space ventilation. } \]

\[
V_o = \text{alveolar ventilation in liters per min. at BTPS.}
\]

Now \( Y = V_a fC \) \hspace{1cm} (10)

and

\[
fC = \frac{y}{V + y - x} = \frac{y}{V + y - \frac{y}{Q}} \hspace{1cm} (11)
\]

Substituting (11) in (10)

\[
Y = \frac{V_o y}{V + y - \frac{y}{Q}} \hspace{1cm} \text{or } y = \frac{QVY}{QV_a - QY + Y}
\]

Putting this value of \( y \) equal to that in equation 4 and solving for \( V_a \)

\[
V_a = \frac{Y(Q + f'C(1 - Q))}{Q(fC - f'C)} = \frac{X(Q + f'C(1 - Q))}{fC - f'C} \hspace{1cm} (12)
\]

Now

\[
X_o = X \frac{273}{310} \times \frac{B - 47}{760}
\]

\[
V_a = 1000 V'_o
\]

\[
fC = \frac{pC}{B - 47} \hspace{1cm} \text{and } f'C = \frac{p'C}{B - 47}
\]
Substituting these values for $X$, $V_a$, and (in the denominator) $f'C$ and $f'C$ in equation 12 we have

$$V_a' = \frac{0.864X_0(Q + f'C(1 - Q))}{pC - p'C}$$

(13)

The equation is more useful when expressed in terms of $X_0$ because this may reasonably be assumed to remain constant while $Y_o$ varies with the rate of breathing. While this equation expresses the ventilation in terms of CO$_2$ tension it can also be expressed in terms of O$_2$ tension by the same method. Thus in equation (11)

$$fC = \frac{y}{V + y - x} = \frac{Qx}{V + Qx - x}$$

(14)

This value of $fC$ is substituted in (10) and solved for $x$. The resulting equation is combined with (3) and solved for $V_a$ to obtain

$$V_a' = \frac{0.864X_0(1 - f'O(1 - Q))}{p'O - pO}$$

(15)

If $f'O = 1$, then $f'C = 0$ and from (15)

$$V_a' = \frac{0.864X_0Q}{p'O - pO}$$

(16)

If $f'C = 0$ then from (13)

$$V_a' = \frac{0.864X_0Q}{pC} = \frac{0.864Y_0}{pC}$$

(17)

If $Y_o = 315$ cc. CO$_2$ per minute equation 17 is identical with equation 2. Thus, when the CO$_2$ output is assumed constant and $p'O = 0$, the lines of constant ventilation of the $pCO_2$-$pO_2$ chart are parallel to the $X$ axis for all altitudes, according to the scale shown in figure 1.

6. Oxygen isoventilation lines (when $X_o$ is constant). Equations 13 and 15 express the alveolar ventilation in terms of the R.Q. On a chart like that in figure 2 they would permit the calculation of the alveolar ventilation which would be necessary to reach a given $pCO_2$ value (equation 13) or a given $pO_2$ value (equation 15) on a specified R.Q. diagonal at any altitude. In order to draw an isoventilation line on such a chart it is necessary to combine these two equations with the elimination of $Q$. This results in the following

$$V_a' = \frac{0.864X_0(1 - f'O - f'C)}{(1 - f'C)(p'O - pO) - f'O(pC - p'C)}$$

(18)

Or, rearranging,

$$pO = p'O + \frac{f'O p'C}{1 - f'C} - \frac{0.864X_0(1 - f'O - f'C)}{V_a'(1 - f'C)} - \frac{f'O p'C}{1 - f'C}$$

(19)
In equation 19 all the terms refer to the composition of the inspired air at a
given altitude except \( pO \), \( pC \), \( X_o \) and \( Va' \). The equation gives therefore the
composition of the alveolar air which would be expected from a given set of con-
ditions if the rate of oxygen consumption and the alveolar ventilation are known.
If \( pO \) and \( pC \) are the only variables then this is the equation of a straight line of
reciprocal slope equal to \(-\frac{f'O}{1-f'C}\) (in the last term) and the intercept on the X
axis is given by the first 3 terms on the right side of the equation.

When \( CO_2 \) is absent from the inspired air, equation 19 may be simplified to
\[
pO = p'O - 0.864 X_o (1 - f'O) \frac{1}{Va'} - f'OpC
\]
When in addition, \( f'O = 1 \) the equation becomes \( pO = p'O - pC \). This means
that in pure \( O_2 \) the alveolar air is always somewhere on this diagonal regardless
of the values of \( Va' \), \( X_o \), or \( Q \).

A set of isoventilation lines plotted from equation 20 for a subject breathing
air at 20,000 feet is shown in figure 3. All the lines are parallel and have a
negative slope of 0.209 which is the fraction of oxygen in the inspired air \( f'O \).
The line for \( Va' = \infty \) is seen to coincide with the line for \( Q = \infty \).

The beneficial effect of high R.Q. due to overventilation at altitude is well
illustrated by figure 3. Point A indicates that a subject at this altitude with a
rate of oxygen consumption of 315 cc. per min. and an R.Q. of 0.8 who is breathing
so as to provide 5 liters per minute of alveolar ventilation will have an al-
veolar \( pCO_2 \) of 42 mm. (This is slightly higher than normal but will illustrate
the principle.) If now the alveolar ventilation is increased to 10 liters per minute the
alveolar \( CO_2 \) tension will decrease and the R.Q. will increase possibly
to 1.4 so that the alveolar air will be represented by point \( B \), the \( pO_2 \) being 22
mm. greater than at \( A \). If the same ventilation rate is maintained for a half
hour or more the R.Q. will gradually fall as excess \( CO_2 \) is blown off until it
returns to the normal value of 0.8 as represented by point \( C \).

It is important to note that this temporary gain in \( pO_2 \) due to hyperventilation
is largely dependent upon the presence of nitrogen in the air. If pure \( O_2 \) had
been inhaled the point \( B \) would have been on the same diagonal as points \( A \) and \( C \)
(i.e., the \( Q = 1 \) diagonal) each point being at the same \( pCO_2 \) level as in figure 3;
and the gain in \( pO_2 \) would have been correspondingly less as shown in figure 4.
The greater the oxygen percentage, the more nearly the R.Q. diagonals and the
\( O_2 \) isoventilation lines coincide with the \( Q = 1 \) line and the less the temporary
gain in \( pO_2 \) with hyperventilation.

Another way to explain this R.Q. effect is as follows. When pure \( O_2 \) is in-
haled the \( CO_2 \) which escapes into the lung must displace \( O_2 \) in equal amounts
so that a 10 mm. increase in \( pCO_2 \) leads to a 10 mm. decrease in \( pO_2 \). When
air is inhaled however an increase of 10 mm. \( pCO_2 \) in the lungs leads approxi-
mately to a decrease of \( pN_2 \) of 7.9 mm. and a decrease of \( pO_2 \) of only 2.1 mm.
The slope of the \( O_2 \) isoventilation lines in figure 3 indicates a loss of 2.1 mm. of
\( pO_2 \) for a gain of 10 mm. of \( pCO_2 \). Thus when overventilating with inspired
air the resulting high R.Q. and increased CO₂ output has the advantage that it maintains the pCO₂ at a comfortable level without appreciably diminishing the gain in pO₂.

Or conversely, it may be said that in air, the same gain in pO₂ is obtained with less increase in ventilation than in pure O₂. After the high R.Q. due to anoxic hyperventilation has returned to normal (point C, fig. 3) there is little difference between pure O₂ and air in this respect. For a fall of 10 mm. in the pCO₂ there is a (permanent) gain of 10 mm. in pO₂ when pure oxygen is inhaled and a gain of about 12 mm. pO₂ in air if the R.Q. is 0.8. It is well known that this initial R.Q. effect is a very important factor in the maintenance of consciousness for short periods at extreme altitudes in air. The principle is not new but is here demonstrated in a novel way.

Fig. 6. Lines of equal alveolar ventilation at three different values of f'O or oxygen fraction in the inspired air. The right side of the figure represents the situation when air is inhaled, the left side, the case for pure oxygen, and the family of lines in the middle represents 80 per cent oxygen. (See text.)

It has been shown in figure 4 that the R.Q. diagonals approach the Q = 1 line when f'O approaches 1.0. The same is true of the oxygen ventilation lines calculated from equation 15 or 19 when X₀ is constant. This result is illustrated graphically in figure 6 where 3 sets of diagonals are plotted for f'O = 0.21, 0.8 and 1.0. These are plotted arbitrarily at different p'O₂ values but this makes no difference because the slopes of the lines are not influenced by the absolute pO₂ but only by the O₂ percentage, f'O as shown by equation 20. In this graph the points of intersection of the iso-ventilation lines and the diagonal Q = 1.0 are marked by circles. For the same ventilation rate these intersections are all at the same pCO₂ value at all altitudes. Thus as f'O approaches 1.0 these iso-ventilation lines rotate anticlockwise about these marked intersections until at f'O = 1.0 in pure oxygen they coincide with the Q = 1.0 diagonal.

7. Carbon dioxide isoventilation lines (i.e., when X₀ is constant). The isoventilation lines shown in figure 3 were derived on the assumption that the rate of oxygen consumption is constant. Passing along one of these lines from low
to high CO₂ tensions it is found that the pO gradually diminishes as the R.Q. increases. The reason for this is that at the higher R.Q. values the rate of CO₂ output is greater for the same amount of O₂ taken out. As more CO₂ comes into the lungs from the blood per minute the volume of air that comes into the lungs from the trachea is correspondingly diminished because there is less decrease in volume due to oxygen absorption. Hence the oxygen in the alveoli is not renewed so rapidly and the oxygen tension falls.

When the rate of carbon dioxide output is assumed to be constant a very different set of isoventilation lines results. This can be shown by replacing

\[ \frac{X_o}{Y_o/Q} \text{ in equations 13 and 15, and then combining them as before with the elimination of } Q. \]

This results in the following

\[ V'_a = \frac{0.864 Y_o(1 - f'C - f'O)}{(p'C - p'O)(1 - f'O) - f'C(p'O - pO)} \]  

or, rearranging

\[ pO = p'O + \frac{f'C(1 - f'O)}{f'C} + \frac{0.864 Y_o(1 - f'C - f'O)}{V'_a f'C} - \frac{1 - f'O pC}{f'C} \]  

Evidently these equations are very similar in form to equations 18 and 19. When \( f'O = 1 \) or when \( f'C = p'C = 0 \) equation 21 reduces to equations 16 or 17.

The two sets of isoventilation lines given by equations 18 and 21 may be called for convenience the oxygen and the carbon dioxide isoventilation lines respectively. The relation between them can be shown most simply when there is no CO₂ in the inspired air as illustrated in figure 7. In this graph R.Q. diagonals for \( Q = 0.5, 1.0 \) and \( 2.0 \) are drawn for two different altitudes breathing air. The oxygen isoventilation lines are drawn from equation 20 assuming that \( X = 300 \text{ cc. per min.} \) and the carbon dioxide isoventilation lines, parallel to the \( X \) axis, are drawn from equation 17 assuming that \( Y = 300 \text{ cc. per minute.} \)
When $Q = 1.0$ both $Y_o$ and $X_o = 300$ and both the $O_2$ and $CO_2$ 5 lit/min ventilation lines intersect at both altitudes on the $Q = 1.0$ lines. The same is true for the 10 and 20 lit/min lines. When however $Q = 2.0$ the 10 lit/min $O_2$ line intersects the 5 lit/min $CO_2$ ventilation line. But this is a 5 lit/min line only when $Y_o = 300$. Since $Q = 2.0$, $Y_o$ in this case = 600 so the ventilation is really 10 lit/min at this point if $X_o = 300$ as indicated by the $O_2$ line. Conversely these ventilation lines may be regarded as lines of equal $Y_o$ or $X_o$ if the alveolar ventilation = 10 lit/min. The numerical values appropriate to this interpretation have been inserted in the figure. Thus the two 600 cc/min lines intersect on the diagonal $Q = 1.0$ but the 300 cc. $X_o$ and the 600 cc. $Y_o$ lines intersect on the $Q = 2.0$ diagonal.

8. Ventilation when inspired air contains $CO_2$. When the inspired air con-

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**Fig. 8.** The effect of adding $CO_2$ and $O_2$ to the inspired air. Three sets of R.Q. diagonals are shown for three different mixtures. All are calculated according to equation 6 for an altitude of 29,815 feet. The isoventilation lines traversing the R.Q. diagonals are calculated according to equation 22 assuming that the $CO_2$ output, $Y_o = 315$ cc per minute. As the $pCO_2$ increases in the inspired air these lines deviate from the horizontal. Figures on these lines indicate the alveolar ventilation in liters per minute.
tains a certain amount of CO₂ the carbon dioxide ventilation lines plotted from equation 22 are not parallel to the X axis but they have a reciprocal slope equal to \(-\frac{1-f'O}{f'C}\). This is illustrated in figure 8 where three sets of R.Q. diagonals are represented for different mixtures of O₂ and CO₂ at 29,815 feet. The effect of adding 20 mm. of CO₂ to the inspired air is to raise the origin of the R.Q. diagonals by 20 mm. Breathing CO₂-free air in the lower left the CO₂ isoventilation lines are parallel to the X axis as in figures 6 and 7 and are drawn from equation 17 assuming that \(Y = 315 \text{ cc/min.}\) In a 10.5 per cent CO₂, 31.4 per cent O₂ mixture the isoventilation lines have a slight slope to them and this is more marked when the oxygen and the CO₂ are both still further increased. It may also be observed that as the nitrogen percentage decreases, the R.Q. diagonals come closer together as they did in figure 4 for oxygen alone. When the nitrogen percentage = 0 the CO₂ isoventilation lines coincide with the \(Q = 1\) diagonal just as the O₂ isoventilation lines do. This can be seen from equation 22. When \(f'O = 1\), then \(f'C = 0\) and this equation reduces to \(p'O = p'O - pC\) which is the equation for the \(Q = 1\) diagonal.

9. Advantages of inspired CO₂ at altitude. It has been shown that a high R.Q. is advantageous at altitude because it permits a high ventilation rate and consequently a high alveolar \(pO₂\) without serious hypocapnia. The same is true of CO₂ added to the inspired air. The benefit in other words is the same whether the CO₂ is exogenous or endogenous.
The advantage gained from adding CO₂ to the inspired air is shown in figure 9. Two sets of altitude diagonals are drawn for 20,000 feet. The lower set applies to the case where CO₂-free air is breathed. If now CO₂ is added to air at this altitude until the CO₂ tension is 20 mm., the CO₂ percentage will be 6.62 per cent and the oxygen will have been diluted to 19.5 per cent. The total dry pressure \(B - 47 = 302\) mm. The broken lines in figure 9 are O₂ isoventilation lines drawn from equation 19. With this particular CO₂ mixture made by diluting air the equation for the ventilation is the same whatever the amount of CO₂ added but this is a special case selected both because of its simplicity and because it is the most practical way of producing such a mixture for experimental purposes. If a subject goes to 20,000 feet without changing his rate of CO₂ output or his ventilation he would have an alveolar air composition represented by point A (assuming an R.Q. of 0.8). Due to the stimulus of anoxia he would increase his ventilation rate from 5.6 to perhaps 11.6 liters per minute and would move approximately to point C where his R.Q. is 1.4. If however he were to use a 6.6 per cent CO₂ mixture he would be at point B for the same rate of ventilation or at about point E if his ventilation were increased to 20 liters per minute. Both points C and E are only temporary because if the ventilation were to continue at the same rate the R.Q. would eventually return to 0.8 and the alveolar air would move to points D and F respectively. Point D would have the higher arterial saturation but it has the disadvantage of a considerable amount of hypocapnia. The performance would certainly be better at F than at D and probably better at E than at C. Point B is better than C because it can be maintained indefinitely but the arterial saturation is only about 67 per cent while at D it is about 90 per cent (see fig. 2 for saturation lines). At F however the saturation is 85 per cent and the pCO₂ is high enough for normal performance. Comparing points A and B it is evident that the addition of 20 mm. of CO₂ has increased the oxygen tension 20 mm. without decrease in the pCO₂. Comparing points B and D it is evident that at the same rate of ventilation the addition of 20 mm. of CO₂ to the inspired air has raised the alveolar pCO₂ 21 mm. with a loss however of 5 mm. of pO₂.

The \(Q = 1\) diagonal for 6.6 per cent CO₂ has been continued as a broken line to the X axis which it intersects at 79 mm. This represents the oxygen tension of the inspired air at an altitude of 15,137 feet. It may be said therefore as an approximation that the addition of this amount of CO₂ has lowered the equivalent altitude 4 or 5 thousand feet.

10. Survival limits on the CO₂-O₂ diagram. Finally it seems worth while to call attention to figure 10 which represents a CO₂-O₂ diagram on a much larger scale to include the lethal limits in all directions. The shaded area up to about 1 atmosphere of pO₂, represents the normal or physiological area. It is bounded on the top by the CO₂ narcosis level at about 50 mm. on the bottom by the acapnia threshold at about 20 mm. pCO₂ and on the left by anoxia at altitudes > 40,000 feet or tensions of O₂ < 100 mm. The corners of this shaded area are rounded off indicating that the effects of anoxia are additive to both high CO₂ and low CO₂. At an air pressure above 5 atmospheres the disorienting effects of high nitrogen concentration become noticeable (Behnke et al., 1935) and at
10 atmospheres or diving depths under water of 200 feet this nitrogen narcosis becomes severe. At 3 atmospheres of \( \text{O}_2 \) pressure oxygen poisoning becomes evident in some 2 hours of exposure and this effect is more severe at the higher CO\(_2\) tensions. Thus high CO\(_2\) is additive in its effects to both high \( \text{O}_2 \) and high nitrogen. The combination of high CO\(_2\) with still higher \( \text{O}_2 \) pressures may result in collapse within a minute as reported by Case and Haldane 1941. Many details of this diagram are still most uncertain as for example the alveolar \( p\text{CO}_2 \) level at high oxygen pressures and the effect of combining acapnia and high nitrogen or high oxygen. It seems a fair guess that these effects will also be additive so that at high pressures acapnia will occur at higher \( p\text{CO}_2 \) levels than

![Diagram of CO\(_2\) Narcosis](http://ajplegacy.physiology.org/)

Fig. 10. An alveolar air diagram to illustrate schematically the relation of the normal range of \( p\text{O}_2 \) and \( p\text{CO}_2 \) to the abnormal regions characterized by too much or too little of either gas. Abscissae represent oxygen tensions in atmospheres. If these same partial pressures of \( \text{O}_2 \) are reached by breathing air at pressures indicated on the middle horizontal scale there is the additional complication commonly ascribed to nitrogen poisoning. The lowest scale on the axis of abscissae represents the corresponding altitude in feet breathing \( \text{O}_2 \) or the depth under water breathing air which would give the same \( \text{O}_2 \) pressure. Exact positions of the lines are not well established.
under normal conditions. Figure 10 is presented only as a simple means of utilizing this diagram for the demonstration of a large number of physiological effects of respiratory gases.

No originality is claimed for any of the physiological facts reported in this paper. The method of presenting the facts and parts of the mathematical treatment are new and bring out the relationships in a clear and quantitative way which should be helpful in both teaching and research.

SUMMARY

Equations are developed for calculating the alveolar air composition in terms of the altitude, the composition of the inspired air, the alveolar ventilation volume, the rate of oxygen consumption and the R.Q. These equations are illustrated graphically in a series of charts in which $pCO_2$ is plotted as ordinates against $pO_2$ as abscissae. The important physiological consequences of these equations are pointed out. The paper represents a theoretical treatment of certain known facts of high altitude physiology.

REFERENCES