If one divides the energy equivalent of the total excess oxygen consumed as a result of a short sprint at top speed by the number of seconds occupied in the run, one finds (after correcting for the start and the pull up) (cf. Sargent, 1926) that the rate of energy expenditure while running at a maximum speed is about 13 horse power for an average man. From measurements on isolated muscles (Hill, 1926) as well as from calculations made by Furusawa, Hill and Parkinson (1927) on sprinters, it may be concluded that about 40 per cent of this energy was expended during the sprint in the "initial" or anaerobic phase of muscle contraction, the remainder representing the inefficiency of recovery. The present paper is concerned with the disposition of this initial energy which is being expended at a rate of about 5.2 horse power.

The conclusions reached and the problems involved are illustrated in the following diagram:

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1 Unpublished measurements from this laboratory for which I am indebted particularly to Dr. E. Fischer, Mr. H. Brody and Mr. C. I. Wright.

2 Furusawa, Hill and Parkinson (1927) have found one man in which this figure was 8.5 horse power and Gertz by a somewhat similar method has estimated 6 to 8 horse power for the expenditure of mechanical energy while running at top speed. These men, however, were fast runners. The average speed of our runners was only 8.2 yards per second. Taking an average weight of 150 lbs. and an average propelling force of 0.75 the body weight the horse power would be $8.2 \times 3 \times 150 \times 0.80 \div 550 = 5.4$ horse power.
Some of the arrows in this diagram should be accompanied by question marks. Thus the source of the work is doubtful, whether from shortening energy or previously developed potential (tension) energy. It is also possible that the “shortening energy” is partly wasted or partly used to overcome friction. The point of chief interest in this diagram, however, is the magnitude of the rate of useful work which has actually been measured from moving pictures and was found to be 2.95 horse power. The various items which are included in this figure are shown in the last column and include work against gravity, changes in velocity due to wind resistance and to bringing the foot into contact with the ground, and the kinetic energy changes of the limbs. Marey and Demeny in 1885 made an estimate of the magnitude of these various components of the work. Their estimates for the first two items agree fairly well with ours but their estimate of the kinetic energy of the limbs was 7 or 8 times too small because they measured only the average velocity of the limbs during a swing, neglected movements around the knee and elbow joints and neglected the back swings (cf. Amar, 1923, p. 504). In walking, the work against gravity is a much more important factor than in running and indeed Benedict and Murschhauser (1915) found it to be 23 per cent of the total oxygen consumption in excess of the standing value. Hill, 1927, who has discussed in many valuable papers the problems connected with rapid movements of muscles has regarded frictional loss as the limiting factor in a fast run and as in fact the only item of considerable importance. From the above diagram, however, it is obvious that, after deducting the work, only 5.2 minus 2.95 or 2.25 horse power is left to be divided between the three items of fixation energy, waste heat and frictional loss. The isometric contractions involved in making the body rigid and in fixating the joints, etc., are by no means negligible although there is no method of measuring them exactly; hence the energy left for frictional loss is not so large as has been supposed. In order to show how significant this conclusion is for the theory of muscular contraction, some further discussion of muscle viscosity is necessary.

A new method of demonstrating muscle “viscosity.” The problem of “muscle viscosity” can be clearly presented as applied to man, by the following simple experiment in which the muscle tension is measured as it decreases with increasing speed of movement. The subject seats himself on a table with one leg hanging over the edge. Arrangements are made for recording variations in the angle of the knee with time as the leg swings from the knee. This is most easily done by fastening the leg to a light wheel, the horizontal axis of which coincides with the axis of the knee joint. As the wheel revolves with the leg it winds up a thread carrying a short

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*The shortening energy is the excess heat developed when a muscle is allowed to shorten (Fenn, 1923). Thanks to Professor Hill’s sense of humor this is better known in the literature as the “Fenn effect.”*
pointer; the thread is held taut by an elastic band. The curve traced by
the pointer on a revolving drum as the leg swings indicates angles plotted
against time. The slope of this curve represents the angular velocity in
radians per second. If the angular velocity is plotted as a function of time
the slope of the resulting graph represents angular acceleration and must
be proportional at any moment to the torque applied to the leg at that
time. The proportionality factor is the moment of inertia of the leg
around the knee axis. This may be estimated with fair accuracy from the
measurements of Braune and Fischer (1892) on cadavers and enables us
to calculate the net external force exerted by the muscles at any moment
in the swing. If the subject endeavors to extend his lower leg when the
knee angle is 90° while the torque is measured with a spring balance it is
found that he can develop a force of 19 kgrm. at a distance of 43 cm. from
the knee axis. If the lever arm of the extensor muscle is 4 cm. this indi-
cates a force of $\frac{43}{4} \times 19$ or 204 kgrm. If while the muscles are exerting this
torque the experimenter suddenly releases the foot, the torque imparts an
acceleration to the lower leg which can be measured from the graphic
record. If the moment of inertia of the leg below the knee be taken as
$3 \times 10^6$ gram and centimeter units, the torque producing the acceleration
can be calculated (torque = moment of inertia $\times$ angular acceleration).
Analysis of a quick release record obtained in this way shows that before
the foot has travelled 2 cm. the force exerted by the extensor muscles
(on a 4 cm. arm) has fallen from 204 to 108 kgrm. and when the foot has
moved 4 cm. the force has fallen to 55 kgrm. When the foot has moved
6 cm. the velocity has become constant; there is no further acceleration
and hence no further external force except that necessary to overcome
friction in the joint.

In a similar manner a record of a free kick with the lower leg was taken.
The angular acceleration was determined from the record and thence
the force exerted at different times was calculated. In this case a maxi-
imum external force of about 142 kgrm. was developed after the foot had
moved about 1 cm. but by the time the foot had moved 5.3 cm. of the
arc the velocity had become so great that a tension of only 37 kgrm. could
be maintained.

Critique of measurements of muscle "viscosity." These results show in a
fairly quantitative way how extremely quickly the tension falls off in a
muscle after it is suddenly released and how small a tension can be main-
tained when a muscle is rapidly shortening. This failure to develop
tension while shortening may, in this case, be due partly to a reflex cessa-
tion of stimulation or a reflex inhibition and there would seem to be every
reason for expecting that such a reflex would occur. On the other hand,
the loss of tension during shortening may be due to some characteristic of
the muscle itself since it has likewise been demonstrated in isolated muscles (Hill, 1926). With this idea in mind it has been called "muscle viscosity." The term implies that some internal rearrangements inside the muscles are necessary before external tension can be displayed and that it is a mechanical resistance to these movements which prevents a muscle from re-developing tension, during shortening, rapidly enough to manifest tension externally. But in this case, the delay in the development of tension might equally well be in some chemical reaction involving the mobilization of the necessary energy for the contraction. In such a case the term "viscosity" would be inappropriate. The fact that the process of developing tension during shortening (Fenn, 1923) necessitates an extra liberation of energy (shortening energy) and that within certain limits the faster the shortening the less the extra energy developed, suggests this latter interpretation. Hartree and Hill (1928) in their most recent communication on this subject have confirmed in the main the finding that shortening under tension involves extra energy liberation and have taken it more seriously into consideration in their theories than heretofore, suggesting that it has an equal share with viscosity in determining the amount of work a muscle may do at different speeds of shortening. For the reasons here suggested it seems advisable to warn against the simple interpretation of the term "viscosity." There seems to be as yet no certain way of determining how quantitatively important "viscosity" or friction may be in muscular movements in man. By "viscosity," I mean a mechanical as opposed to a chemical delay in the external manifestation of tension.

In 1922 Hill published an important paper on the work of human arm muscles, contracting against different equivalent masses. With a small equivalent mass the contraction was rapid and little work was done and vice versa. It was found empirically that the diminution in work with increasing speeds was proportional to the speed of shortening and the interpretation was suggested that this energy which failed to appear as work was developed as potential energy but degraded into heat in overcoming frictional resistance in the muscles. While there is doubtless some truth in this interpretation, the fact that isolated muscles liberate less heat per second (i.e., for a given duration of stimulus) when shortening rapidly than when shortening slowly, makes it probable that in rapid contractions of the arm muscles less energy per second is actually developed. Hence we are at a loss to know how much of the diminution of tension with high speeds is due to diminished rate of energy expenditure and how much is due actually to frictional loss.

More recently in his experiments at Cornell, Hill (1927) (and Furusawa, Hill and Parkinson, 1927) has made use of a similar idea in studying the equation of motion of a sprint runner (cf. also Gertz, 1929). The runner
gradually accelerates until he reaches a constant maximum velocity. It is shown experimentally that his velocity as a function of time can be quantitatively explained by supposing that he is being propelled by a constant force and being resisted by a force which is proportional to his speed. This resisting force is again muscle viscosity (or "something which behaves like viscosity"). There can be no question that the equation derived on these assumptions adequately fits the facts. By means of this equation it is possible to determine the magnitude of the hypothetical propelling force; this force multiplied by the distance travelled gives the work done in the run. Since no appreciable external work is done in the run it is supposed that "the whole of the mechanical energy liberated is used in overcoming the frictional resistance of the body itself, particularly the 'viscosity' of the muscles themselves" (Hill, 1927).

This method resembles the thermodynamic method; it gives us the end results without telling us the mechanism. Assuming that it tells us correctly the total amount of mechanical energy expended it does not tell us in what way it was expended. It certainly is not all expended in overcoming viscosity. In the sentence following the one quoted above, Hill (1927) mentions the kinetic energy of the arms and legs which must alternately be created and destroyed, and uses it to show why the cost of running increases so rapidly with the speed. He does not emphasize, however, that this kinetic energy must be created in spite of viscosity and that a high viscosity would assist in destroying it, rather than the reverse. In fact as already mentioned, over one-half of the work as measured by Hill's method is actually used in creation and destruction of kinetic energy in the arms and legs, in changes in velocity of the whole body and in work against gravity.

There is another assumption underlying Hill's equation expressing the motion of a runner; the propelling force is assumed to be constant. The only justification for this is that the runner is making a maximal effort throughout the run. At the start however, the limbs are moving slowly and it seems likely that the force exerted (including the internal force used in overcoming viscosity) might be greater than at the end of the run when the limbs are moving at maximum speed.4

In case the possible inconstancy of the propelling force does not invalidate Hill's calculations we are left with the difficulty of explaining how it is possible that the items described as "useful work" should "behave like viscosity" in being proportional to the velocity of the runner. This must

4 Isolated muscles when stimulated with a constant stimulus give off less heat when shortening rapidly under low tension than when shortening slowly under higher tension. (Hartree and Hill, 1928.) This suggests that in rapid running in man less energy is liberated per second and hence less tension exerted. The experiments are not however exactly comparable.
be the case or Hill's equation would not fit the facts as it apparently does. One would expect that the kinetic energy of the arms and legs for example would vary as the square of the velocity. An increase of velocity is attained however by increasing the length of the stride, as well as by moving the arms and legs more rapidly. Hence it is possible that there is a considerable range over which this factor may increase so nearly in proportion to the velocity as not to invalidate Hill's equation. Hill's facts therefore lend themselves admirably to the interpretation which he has put upon them but it does not seem certain that they do not lend themselves to some other interpretation equally well.

The remainder of this paper is devoted to the measurement of the work of acceleration and deceleration of the limbs. The measurement of the work against gravity and the velocity changes will be described in a later paper.

**The kinetic energy of the limbs. Method of measurement.** The method used was the same as that originally used by Marey and Demenceny, i.e., the moving picture. Due to the connection of one of us (C.A.M.) with the Eastman Kodak Company and particularly with the work of making the Eastman Teaching Medical Films, this work was much facilitated. In order to be able to make accurate absolute measurements from the film it was necessary to have the runners run behind a white wooden lattice work making a coordinate system with squares 1 meter on a side.

Figure 1 will show the general arrangement used for this purpose. A telephoto lens was used in taking the photographs so that the camera could be placed 30 meters from the runners. If the runners ran ½ a meter behind the lattice work, the error in measuring their horizontal velocity from their positions in relation to the lattice was 1.7 per cent. This correction has been neglected in the calculations. The sharpness of the image on the film depends upon the brevity of the exposure. In the first film taken the exposure was about 0.003 second, but in the second film this was reduced to about 0.001 second with a corresponding improvement in definition.

**Timing:** Since the camera is turned by hand during the exposure the speed of the film is not known nor is it entirely regular. In order to time the pictures, wooden balls (croquet balls), 4 inches in diameter, were dropped in front of a vertical scale so that they appeared at the side of the film. The scale was graduated in tenths of a second so that the speed of the film could be readily determined. One of these balls is seen falling in figure 1.

**Subjects:** Two films were taken, one in May, 1928 and one in October, 1928. In the first case 19 men made each one run in front of the camera. In the second case 15 men ran each three times. The men ran one behind another as close together as convenient. Except in one or two cases they were all instructed to run at top speed, without any special sprints.
Fig. 1. Reproductions from strips of the film showing a runner behind the lattice from which measurements were made.
Wallace O. Fenn

as they passed in front of the camera. In the first case they ran on a concrete walk and in the second case on the level turf. The general arrangement of the two films is shown in figure 1, the first 3 rows being taken consecutively from the second film and the last row from the first film. In both cases the men were members of some of the regular classes in physical education at the University of Rochester. We are much indebted to Doctor Fauver and his colleagues at the gymnasium for their cooperation in making these runners available for us. With the exception of one first rate sprinter (pictured in fig. 1) in the second group, none of the men was selected in any way. They may be regarded as a fairly representative group of college undergraduates. In order to provide fixed points for measuring the successive positions reached by a runner the men in the second group were provided with markers. The man in figure 1 shows the nature of these. One was a white cloth tied around the neck with a black spot on it. The other was a circular white tag, carrying a black spot which was supported on a stiff wire frame tied firmly around the waist in such a way that no movement in relation to the body was possible.

Measurement of the films: For purposes of projection and measurement an improvised lantern was used. In the preliminary measurements it turned out that the chief source of error lay in the buckling of the film. To avoid this the film was passed between two glass plates which could be clamped tightly together when the desired frame was brought in front of the lens. The film used was the standard 1½ inch cinemagraph film. When projected, one meter on the original lattice behind which the men ran measured 16.6 cm. For the purposes of the data to be described in this paper, measurements were made of the angles of the upper and lower arms and of the upper and lower legs. It proved to be unnecessary to measure every picture in most cases but every other picture was measured. For other purposes the vertical and horizontal positions of the markers were also determined. Where markers were absent the tip of the nose was chosen as the most definite point. It was found that the angles could be estimated within 2 degrees almost without exception. A protractor was merely laid along the longitudinal axis of the image of the limb in question and the angle read off from the intersection of a plumb line hanging from the center of the protractor.

Calculation of the kinetic energy: For purposes of calculation it is considered that the runner is standing still, as on a treadmill, and is waving his arms and legs as illustrated on the film. This method has the advantage that the kinetic energy turns out to be high in that limb where the work is being done. If the kinetic energy is calculated in relation to the ground, then the limb going backwards has very small kinetic energy although the actual effort on the part of the runner is as great in pushing it backwards as in pushing it forwards. Both methods of course lead to the
same result as far as the kinetic energy of the whole runner at any moment is concerned. But the distribution of energy over the body depends upon the point of reference chosen. The use of the runner himself as a reference point has another advantage which will be discussed later; it minimizes the calculated transmission of energy from one moving part to another. It also eliminates the necessity of knowing the momentary forward velocity of the common center of gravity of the body with great accuracy.

The kinetic energy of any part of the body with relation to the body depends upon its translational velocity with relation to the body. To this must be added its energy of rotation. Let the suffix 0 refer to the body as a whole and the suffixes 1, 2, and 3 refer respectively to the trunk, the upper leg (or arm) and the lower leg (or arm). Let m represent the weight and ω the angular velocity, v the linear velocity and s the distance to the center of gravity of the part in question and k its radius of gyration around its center of gravity. s is measured from the hip joint in the case of the upper leg and from the knee in the case of the lower leg and similarly, *mutatis mutandis* for the arm. Then the kinetic energy of the body as a whole at any moment will be (cf. Fischer and Steinhausen, 1925):

\[
\frac{m_0 v_0^2}{2} + \frac{m_1 v_1^2}{2} + \frac{m_1 \omega_1^2 k_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_2 \omega_2^2 k_2^2}{2} + \frac{m_3 v_3^2}{2} + \frac{m_3 \omega_3^2 k_3^2}{2} + \text{etc.} \quad (1)
\]

Here \(v_0\) in the first term represents the velocity of the center of gravity in relation to the ground and is neglected for our present purpose. The variations in \(v_0\) will be discussed in a later paper. \(v_1, v_2\) and \(v_3\) represent velocities in relation to the common center of gravity of the whole body. The values of \(m\) are calculated from the weight of the runner according to the factors determined by Braune and Fischer (1894) on cadavers. Thus if the weight of the whole runner is 1.00 the weights of the limbs, \(m\), are as follows:

<table>
<thead>
<tr>
<th>Part</th>
<th>m</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper arm</td>
<td>0.0336</td>
<td>0.47</td>
</tr>
<tr>
<td>Lower arm plus hand</td>
<td>0.0312</td>
<td>0.66</td>
</tr>
<tr>
<td>Upper leg</td>
<td>0.1158</td>
<td>0.44</td>
</tr>
<tr>
<td>Lower leg plus foot</td>
<td>0.0705</td>
<td>0.61</td>
</tr>
</tbody>
</table>

This table also gives values of s in fractions of the length of the limb. The lengths of the limbs were measured from the photographs. The second group of runners was asked to pose for this purpose with the arms and legs conveniently bent and in a plane perpendicular to the camera for greatest accuracy of measurement. The limbs were measured from joint to joint. The lower arm was measured from the elbow to the wrist and the lower leg from the knee to the lower extremity of the tibia. The radius of gyration, \(k\), according to the measurements of Braune and Fischer (1892) on cadav-
vers may be taken with fair accuracy as 0.3 of the length of the limb. For this purpose we have to take account of the length of the hand and of the foot. Consequently we have added 13 cm. to the length of the lower arm, as measured above, this being the distance from the wrist to the first interphalangeal joint and 6 cm. to the length of the lower leg, this being the "height" of the foot as given by Braune and Fischer (1892). If therefore the length of the lower arm from the elbow to the wrist is 26 cm. the center of gravity of the lower arm plus hand is located $0.66 \times 26$ or 17 cm. below the elbow and the radius of gyration around this center of gravity is $(26 + 13) \times 0.3$ or 11.7 cm. It was not found possible to measure the lengths of the limbs with great accuracy from the photograph.

![Fig. 2. Diagram of body and leg](http://ajplegacy.physiology.org/)

Our most careful measurements failed to distinguish any constant difference between the lengths of the upper and lower limbs either in the arms or the legs. Thus the length of the lower arm from the elbow to the wrist is taken as equal to the length of the upper arm from the shoulder to the elbow. Likewise the length of the lower leg from the knee to the ankle is taken as equal to the length of the upper leg from the hip to the knee. All of these measurements are approximations but the errors so made cannot appreciably affect the order of magnitude of the results obtained.

In order to calculate the kinetic energy from equation (1) it is, therefore, necessary to measure from the film only the value of the angle $\alpha$. From this the values of $v$ are calculated in the following manner. Figure 2A represents $m_i$ and $m_2$, the latter having an angular velocity of $\omega_2$ and the
angular rotation of \( m_1 \) being neglected. Its linear velocity in relation to \( m_1 \) is therefore \( v_1 \omega_1 = v_2 \), as represented. Its energy of translation with respect to \( m_1 \) is therefore \( \frac{m_2 v_2^2}{2} \) and its rotational energy, around its center of gravity is \( \frac{m_2 l_2^2 \omega_2^2}{2} \). The sum of these two factors is of course equal to \( \frac{I_2 \omega_2^2}{2} \) where \( I_2 \) is its moment of inertia around the hip joint.

The calculation of the kinetic energy of the lower leg is not quite so simple (fig. 2 B). One first determines the linear velocity of the knee joint, which may be called \( v_1 \) where \( l \) is the length of the upper leg. \( v_1 = l \omega_2 \). Now if the angular velocity of the lower leg, \( \omega_2 = \alpha \), i.e., if the angle of the lower leg with relation to the horizontal does not change, then all parts of the lower leg, including its center of gravity will move with a velocity \( v_1 \) as indicated. But if at the moment at which the knee joint moves with a linear velocity \( v_1 \), the lower leg is also changing its angle with the horizontal with an angular velocity \( \omega_2 \), then the true linear velocity of the center of gravity of \( m_3 \) will be the resultant of \( v_1 \) and \( \omega_3 s_3 \). This resultant is \( v_3 \). The actual determination of \( v_3 \) can be done most rapidly by graphical methods rather than by trigonometry. One simply chooses a point of origin on a sheet of coordinate paper and lays off the components in their proper directions and measures the resultant with a millimeter rule. It is convenient for this purpose to let 1 cm. represent a velocity of 100 cm. per second. Knowing \( v_3 \), the kinetic energy of the lower leg is calculated by the usual formula,

\[
\frac{m_3 v_3^2}{2} + \frac{m_2 l_2^2 \omega_2^2}{2}
\]

A word should be included about the evaluation of the angular velocities. The data actually obtained are the angular positions of the various members at every other frame (exposure) on the film, or at every 0.016 second approximately. These angles are plotted out against the frame number (time) so obtaining displacement curves. These curves are smoothed out graphically with care to preserve all the significant variations. Such displacement curves for the upper and lower arms and the upper and lower legs are shown in figure 4. The angle which is measured and which is plotted in figure 4 is the angle \( \alpha \) illustrated in figure 2, i.e., it is the angle made by the limb with the horizontal line in front of the runner. This convention has the advantage that when \( \alpha \) is increasing the leg is going backwards and vice versa. The displacement curves are drawn for one complete cycle. The length of the cycle can be determined either from the contour of the curves or from the interval elapsing between foot-contacts with the ground. The moment when the toe leaves the ground can be
determined with somewhat more accuracy than the moment when the foot touches the ground, and this point for the two feet is preferred for determining the cycle length. Care is taken in smoothing out the displacement curves to see that they begin and end at the same angle. Since a runner does not always run with perfectly regular rhythm this sometimes involves a slight violation of the observed data but this change is altogether immaterial and doubtless represents just as fair an average cycle for that runner as the actual movement observed. It is convenient to have the two ends fit in this way in subsequent parts of the analysis.

From the smoothed displacement curves the slopes are read off by a straight edge laid tangent to them at every fourth frame, or every 0.03 second approximately. It was found unnecessary to carry through these laborious calculations for every frame measured. With the angular velocities so determined the remainder of the calculation was carried out as indicated.

Now strictly speaking, equation (1) calls for the velocities of the various members of the body in relation to the common center of gravity of the whole body. Then if one can measure the velocity, \( v_0 \), of this common center of gravity one can determine the total kinetic energy of the whole moving system. It is not particularly easy to determine precisely the velocity of the common center of gravity because, as will be shown later, the body rocks backwards and forwards to some extent in running and because the position of the common center of gravity inside the body changes with the changing positions of the limbs. The effect of the latter factor can be determined and will be discussed later. It should be noted, however, that the method of calculation given above determines actually the kinetic energy of the legs in relation to the hip joint and the kinetic energy of the arms in relation to the shoulder joint. Besides this, it does not take account of the relative movements of the two shoulder joints. When the arm moves forward the shoulder joint moves forward with it. We have attempted in the case of one runner to take account of this factor by measuring directly from the film the movements forward and backwards of the shoulder joint itself. In this way it is possible to estimate that an allowance for this factor, in the case of the arms, would increase the calculated kinetic energy of the arms about 30 to 50 per cent. The effect due to the movement of the hips would be much less than this. Since the kinetic energy of the arms is rather small in comparison to that of the legs anyway, the total error involved in this factor is not large and tends to make the true figure larger (perhaps 10 per cent) than the calculated, rather than the reverse.

We have also endeavored to make allowance for the movements of the center of gravity within the body due to the varying positions of the limbs and in a number of cases we have corrected our results for the movements
KINETIC AND FRICTIONAL FACTORS IN WORK OF SPRINTING

of the common center of gravity within the body. Allowance for this factor sometimes makes the observed kinetic energy at any particular moment larger and sometimes smaller but in any case the difference is not large, the velocities with which the center of gravity is shifting inside the body being only about \( \frac{1}{10} \) of the velocities with which the limbs are moving.

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Fig. 3. Data for the left leg of runner 11. The lower graphs show the kinetic energy of the upper and lower left legs in kg. m. as function of time. The middle graphs show the angles between the leg (upper or lower) and the horizontal. The dotted portions indicate the beginning of another cycle. The arrows in the uppermost diagram show the magnitude and direction of the velocities of movement of the upper and lower leg and the resultant \( \mathbf{v} \).

RESULTS. The complete data for the left leg of one of our runners (no. 11) chosen at random, are given in table 1 and are plotted in figure 3. The first column of this table gives the number of the frame on the film.
(We have in all nearly 2000 frames each of which has been numbered on the margin in India ink for purposes of identification.) The second column gives the time in seconds as determined from the rate of fall of the croquet balls. Columns 3 and 4 give the angles with the horizontal of the upper and lower legs (femur and tibia) as measured from the film. The slopes of the displacement curve drawn through these points, as graphically determined, are given in columns 5 and 6 in degrees per frame. Columns 7 and 8 are calculated from column 5 by changing to radians per second and multiplying by \( s_2 \) and \( l_2 \) respectively. Column 9 is calculated similarly from column 6 for the lower leg. Column 10 is the resultant \( (v_R) \) of the velocities of columns 8 and 9 which are laid off in directions perpendicular to the angles recorded in columns 3 and 4. Columns 11 and 14 are calculated directly from columns 7 and 10 respectively and represent the translational kinetic energy of the upper and lower legs. Columns 12 and
are calculated from columns 7 and 9 and represent the rotational energy of the upper and lower legs respectively. Column 13 is the sum of columns 11 and 12 while column 16 is the sum of columns 14 and 15.

The values of the kinetic energy of the upper and lower legs respectively are given in columns 13 and 16 as they vary with time. These values are plotted in figure 3 in the lower two graphs. The corresponding positions of the leg are shown in the same figure. The graph begins as the left leg starts its backwards movement. The kinetic energy of the upper leg is seen to increase slightly until the moment when the foot makes contact with the ground. (At this moment of contact the foot is moving backwards in relation to the body but forwards slightly in relation to the ground.) Contact with the ground causes a slight check to its backwards movement and the kinetic energy falls off slightly. There is a corresponding irregularity at this point in the graph showing the angle between the upper leg and the horizontal. These angles can be measured with an accuracy of 2 degrees so that slight deviations are significant. The kinetic energy then increases again to a still higher level which is reached near the end of the backward stroke, at which point the kinetic energy falls again to zero. During the succeeding forward stroke the kinetic energy passes again through a maximum.

The kinetic energy of the lower leg never reaches zero but it starts at a low level as the leg starts backwards. At the point where the foot makes contact with the ground there is a slight hump in the curve, the maximum being reached toward the end of the period of foot contact with the ground. When the thigh starts forward the kinetic energy of the lower leg passes through a minimum but does not become zero because of the flexion of the knee. As this flexion continues, the knee being simultaneously carried forward, the kinetic energy of the lower leg passes through a second maximum which declines as knee flexion gives way to knee extension. As the thigh reaches its extreme forward position the lower leg tends to be thrown rapidly forward and downward thus producing a third maximum in the kinetic energy curve.

As already explained the velocity of movement of the center of gravity of the lower leg depends upon the velocity with which the knee carries it (without change in its angle with the horizontal) and the velocity with which its angle with the horizontal is changing. These two vector quantities are represented in the upper part of figure 3. Each point is represented by a jointed arrow in two parts. The starting point of each arrow represents the time to which it applies. The first joint represents the direction and velocity of movement of the knee. The second joint, terminating in the arrow head, represents the direction and velocity with which the center of gravity of the lower leg is moving because of its change of angle with the horizontal. The resultant of these two vectors is $v_3$ as
indicated on the diagram. It will be noticed that each of these vectors is drawn at right angles to the actual position occupied by the limb in question as illustrated below in figure 3. Also it will be noticed that where $v$ is large the kinetic energy of the lower leg is also large and vice versa. The rotational energy is sufficiently small to be negligible for purposes of this comparison.

Similar data from runner 1 are plotted in figure 4. Here the positions of the arms and the kinetic energy of the upper and lower arms are also represented. The displacement curves for both arms and legs showing the angles occupied in successive moments of time are also plotted. On these graphs the points represent the actual measurements taken from the screen and the curves drawn show the extent to which it is necessary to smooth out these curves before calculating slopes. In the case of this runner every frame was measured although as usual, the kinetic energy was calculated only at every fourth frame. It will be seen that the actual measurements in this case do not quite cover the period of a complete cycle so that a slight extrapolation is necessary. In this runner the kinetic energy of the lower leg reached a high peak and then fell off just as the foot left the ground. This great increase in the kinetic energy is coincident with the vigorous push backward given as the foot leaves the ground. The angular velocity of the upper leg becomes high at this moment also as shown by the sudden rise in its displacement curve at frame 25, figure 4. The low minimum in the kinetic energy reached immediately afterwards is coincident with the cessation of movement of the thigh as it turns forward and the low angular velocity of the lower leg as the ankle extends. The diminution in angular velocity of the lower leg due to extension of the ankle is seen clearly by the flat place on the displacement curve of the lower leg at about frame 25, figure 4.

It is the large distance of the lower leg from the body which makes the work necessary to swing it the most important single item in the total kinetic energy of the limbs. This work can be very appreciably diminished if the knee is flexed as it is when the leg is being brought forward for another step, for in this way the moment of inertia of the leg as a whole is much decreased. Thus, to choose a case at random, it was found that the leg of runner 2 had a moment of inertia of $18.9 \times 10^6$ gm. cm.$^2$ with the leg extended (knee angle 142 degrees) while the moment of inertia was only $5.5 \times 10^6$ gm. cm.$^2$ when the knee was flexed (angle 32 degrees). For this purpose the moment of inertia was calculated from the formula of Braune and Fischer (1892).

$$I = m_2 k^2 + m_3 k^2 + m_2 s^2 + m_3 (l^2 + s^2 - 2 l s \cos \beta)$$

where $\beta$ is the angle between the upper and lower legs at the knee, $l$ is the length of the part and $s$ is measured from above downwards. It should be
noted that this formula cannot be used for calculations of the kinetic energy of the whole leg at any particular moment (using the formula

\[
K = \frac{1}{2}mv^2
\]

for calculating the kinetic energy), where:

- \(K\) is the kinetic energy
- \(m\) is the mass
- \(v\) is the velocity

Fig. 4. Kinetic energy and displacement curves of runner 1
$I\omega^2/2$) because it does not enable one to take account of the movement of the lower leg or arm relative to the upper leg or arm; it assumes the whole limb to be a rigid body. Marey and Demeney (1885) took no account of this difficulty nor apparently of the fact that the angular velocity is not uniform throughout the stroke.

On account of the special importance of the lower leg it is necessary to consider how characteristic the lower leg curves of figures 3 and 4 may be. For this purpose similar data from 10 other runners have been plotted in figure 5, at the top of which the approximate positions of the leg are shown diagrammatically.

All of these graphs of figure 5 represent the kinetic energy changes of the lower leg. They are all arranged so that the moment when the foot leaves the ground comes at frame 30. The moment when the foot comes in contact with the ground varies slightly in different runners. In nearly every case there is an irregularity in the curve at this point. Measurements show also that there is always slight bending of the knee to break the shock as the weight of the body comes on the foot. All of these curves are sufficiently characteristic so that it is possible to tell fairly accurately in each case when the foot is on the ground from the shapes of the curve. Each of these curves shows in general three succeeding peaks which differ somewhat among themselves. (The first of these is the most characteristic and coincides approximately with the period of foot-contact.) The second peak comes as a rule at or a little before the time when the angle made by the lower leg with the horizontal begins to decrease instead of increase (marked by the first arrow on each graph) and also near the point where the forward angular velocity of the thigh is at a maximum. The third of these peaks comes approximately at the point where the thigh reaches its maximum forward position (marked by the second arrow on the graph). The last three of these graphs all come from one runner, no. 21, who was the fastest sprinter in the group, his velocity being 8.3 to 8.5 meters per second.

As a starting point in analyzing these curves and calculating the horse power expended by the runners it may be assumed that each time the kinetic energy of a limb increases there is a corresponding expenditure of energy by the muscles, and each time it decreases, a corresponding amount of energy is dissipated as heat. The total kinetic energy developed during one cycle is therefore the sum of all the increases in kinetic energy as taken from the curves. The sum of all these increases for all the limbs divided by the length of the cycle will give the horse power. Data so obtained from all our runners are collected in table 2. The first column gives the runner's number (no. 3 and nos. 22-35 were not analyzed). Column 2 gives the weight of the runner in kilos. Columns 3 to 6 give the increases in kinetic energy observed in the upper and lower arms during their forward
Fig. 5. The variations of kinetic energy of the lower legs of 10 different runners in 10 different cases. The moment when the foot leaves the ground is made to coincide in each case with frame 30.
and backward movements, respectively. The sum of columns 3 to 6 is given in column 7. Columns 8 to 12 give the corresponding increases in kinetic energy of the legs, upper and lower. Figures given in parentheses

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**TABLE 2**

*Increases of kinetic energy (in kilogram meters) of arm and leg during one running cycle*
above the line indicate extra increases associated with the making of foot contact with the ground which it was felt ought to be included. Such additional increases are shown in figure 5 for runners 13 and 19 but these have not been included in the calculations. The sum of columns 8 to 12 is given in column 13. Column 14 will be discussed later. Column 15 is the sum of columns 7 and 13 and represents the total kinetic energy developed in one leg and one arm during one complete cycle. Column 16 represents the horse power for both arms and both legs together, calculated by multiplying figures of column 15 by 2, dividing by the length of the cycle and changing units. The average rate of energy expenditure in the arms and legs alone is seen to be 1.68 horse power. If the total rate of energy expenditure in the sprint is taken as 13 horse power (from oxygen consumption measurements) then this represents 12.9 per cent.

DISCUSSION.

It has been assumed in the interpretation of table 2 that when the kinetic energy decreases in a limb it is not stored as potential energy but is degraded to heat. If it were stored temporarily as potential energy it could of course reappear as kinetic energy either in some other part of the body or at some other phase of the cycle in the same member. It is at this point in fact that the interpretation becomes particularly difficult. Consider first the storage of energy in some potential form.

The effect of gravity: Energy is of course stored as potential energy whenever a limb is raised. To what extent then does gravity assist in the alternating movements of running and to what extent does it modify the conclusions drawn from table 2?

The original theory of running was that of Weber (1836) who regarded the movements of the legs as simple pendulum movements. It has been conclusively shown however by Braune and Fischer for a man walking that gravity does not explain the movements and that active muscular contractions must occur to produce the accelerations observed (cf. Amar. p. 504). What is true of walking must be even more true of running where the velocities of the limbs are so much greater. Consider for example the time necessary for the leg to fall by gravity from a horizontal position in front of the body to a vertical position under the body. Let the masses of $m_2$ and $m_3$ the upper and lower legs be respectively 7000 and 4900 grams, $l_2 = 44$, $s_2 = 19$, $k_2 = 13.2$ and $k_3 = 15.0$. To simplify the conditions let the whole mass of the lower leg be considered as located at the knee joint instead of hanging vertically from it as in normal running. The moment of inertia of the whole leg is then $14.3 \times 10^6$. The torque due to gravity is $(7000 \times 10 \times 980) + (4900 \times 14 \times 980) = 342 \times 10^6$. The time to fall from the horizontal to the vertical, for a compound pendulum is

\[ t = 10.220.33.5 \text{ on May 1, 2017 http://ajplegacy.physiology.org/ Downloaded from} \]
\[ i = \frac{\pi}{2} \sqrt{\frac{I}{mgl}} \left[ 1 + \frac{k^2}{4} + \frac{9k^4}{64} + \frac{225k^8}{2304} \text{ etc.} \right] \]

where \( k = \sin \frac{90 - \alpha}{2} \) and \( \alpha \) is defined as in figure 2. For the case where \( \alpha = 20^\circ \) this reduces to \( 1.73 \sqrt{\frac{I}{mgl}} \) whence the time for fall is calculated as 0.354 second. The actual time for the corresponding change in position during running is less than half this amount indicating clearly that the muscles must have contributed to the fall.

The effect of gravity needs further consideration. Its relation to the leg movements is of chief importance. In the case of the upper leg, if its mass is 7000 and \( s \) is 19 the energy available when it falls from a horizontal to a vertical position is \( 7000 \times 19 \times 980 = 13 \times 10^7 \text{ ergs} = 13 \text{ joules} \). The actual kinetic energy found in the leg after this fall in 7 different runners was 75, 33, 33, 45, 42, and 46 joules or an average of 43 joules, or over times as much as could be derived from gravity. This 13 joules (1.33 kgm. m.) must therefore be deducted from the kinetic energy of the upper leg during its back stroke.

It should be emphasized that the weight of the lower leg does not serve to pull the upper leg down. Actually the upper leg would fall more rapidly if the lower leg were absent. Likewise the arm will fall to the side from the horizontal position less rapidly if a heavy weight is carried in the hand than if the hand is empty (provided friction, etc., is negligible). Its natural period as a pendulum is thereby increased. The leg is therefore pushed downwards to some extent to make contact with the ground. On the forward stroke of the upper leg the maximum kinetic energy is reached at an angle of 75° with the horizontal. At this point a negligible fraction (4 per cent) of these 13 joules has been restored to the leg in potential form and the remainder may be assumed to come from the kinetic energy it now possesses. Hence the kinetic energy as listed in table 2 requires no correction for gravity at this point.

From table 2 it appears that the upper leg develops a kinetic energy of 3.35 kgm. m. on its backward stroke and a similar amount again on its forward stroke. Of the latter, all is produced from muscular activity while of the former, it appears that 1.3 kgm. m. may be assigned to gravity.

Consider now the effect of gravity on the three peaks of the kinetic energy curve of the lower leg as illustrated in figure 5. When the kinetic energy begins to increase for the first of these peaks the foot is already practically on the ground and remains so throughout this part of the curve. Hence none of its kinetic energy as recorded in column 10, table 2 can have come from the force of gravity. When the toe leaves the ground the lower leg is raised and at the same time its kinetic energy rises to a maximum.
This maximum is reached at about the same time that the lower leg reaches its maximum elevation. Hence the work done against gravity in raising the leg should be added to the kinetic energy produced in this knee flexion (column 11, table 2), which has an average value of 4.33 kgm. m. Measurements show that in the case of runner 1 the center of gravity of the lower leg is elevated 33 cm. during this interval and the work done is therefore 33 cm. \( \times 980 \times 4790 \) gms. or 15.5 \( \times 10^7 \) ergs or 1.58 kgm. m. The total energy expended by the muscles in moving the lower leg at this period of the cycle is therefore 4.33 + 1.58 or 5.91 kgm. m. The next peak in the kinetic energy of the lower leg comes when the thigh reaches its maximum forward position. During the interval between this and the preceding peak, the center of gravity of the lower leg remains practically on a level because, as the knee rises, the ankle falls a corresponding amount. Hence none of the potential energy of position which the lower leg possessed at the point of maximum knee flexion can have been transformed into kinetic energy to contribute to this second peak. The figures in column 12, table 2, require therefore no correction.

On the whole, therefore, corrections for gravity entail a deduction of 1.3 kgm. m. for the upper leg and an addition of 1.58 kgm. m. for the lower leg. Both of these are small amounts and the difference is negligible in comparison to other errors. The effect of gravity on the arms may be neglected as a relatively insignificant item in the total balance sheet for the body.

Storage of energy in tendons and muscles: From the above considerations it is obvious that the storage of energy as potential energy of position and its reappearance as kinetic energy is not an important factor in evaluating the mechanical horse power of sprinting. Is it possible, however, that there could be such storage of energy in stretched tendons and muscles? It seems that the tendons can be dismissed because the actual positions reached by the limbs in swinging are not sufficiently extreme to stretch the tendons without the participation of actively contracted muscles. Likewise the resting muscles could not exert appreciable tensions in the positions occupied by the limbs at the end of their strokes. If the limb were stopped entirely by frictional forces all its energy would be degraded to heat and there would be none to store. Suppose therefore that muscles must contract and exert a tension \( F \) against a moving limb for a time \( t \) such that \( Ft \) equals the decrease in momentum of the limb. In doing so the muscle is stretched and might be supposed to have stored up a certain amount of potential energy. It cannot retain this store of energy, however, without continuous contraction. If it has any potential energy it is continuously losing it at a certain rate and continuously redeveloping it. The balance between those two determines the amount of tension maintained and the energy of maintenance. A muscle may therefore be said to
"charge storage" at a rate which would expend the energy value of the stored energy many times over in a few seconds. In the movements of running, a study of the movies shows that the tension is maintained during the reversal of direction of motion of the limbs for the acceleration is practically constant during this period. Presumably in this case then the maintenance expenditure is less than the cost of redeveloping the tension and the back stroke must necessarily be somewhat quicker if the tension is already developed.

But however that may be, the energy which the muscles save by thus avoiding the necessity of redeveloping a certain tension for the back stroke is no measure of the amount of potential energy corresponding to that tension; which is the question at issue for the present discussion. Nor can potential energy be measured by the work which the muscle will do when it is allowed to shorten (cf. Fenn, 1923).

The area of the length tension diagram has often been regarded as so much potential energy but one can never recover anything like this amount of energy as work (not over 30 per cent), nor is it certain that the work which is recovered actually came from previously developed potential energy and not from other chemical breakdowns taking place during the performance of the work. The excess production of energy observed when work is done favors the latter view. The absence of such excess heat in single twitches has been cited, however, as proof that in this case at least the work must have come from previously developed potential energy (Hartree and Hill, 1928). Although I do not yet feel convinced that this absence of excess heat in single twitches is universally true (since I have observed variations in the initial heat production of muscles with change of load even when stimulated by single twitches under Ringer's solution) nevertheless the result, if true, does not preclude any other hypothesis. One observes simply more energy production during the contraction phase when work is performed, as compared to an isometric contraction, and correspondingly less during relaxation. It may be said that the performance of work accelerates part of the chemical breakdown which otherwise does not appear as heat until relaxation. We have no real means of knowing where this same relaxation heat was during the isometric contraction phase—perhaps still in the form of chemical potential energy. Moreover the division of the heat into contraction and relaxation phases is not altogether precise and in any case the relaxation heat is not over half the length-tension area corresponding to the isometric tension developed (Hartree and Hill, 1928). In short many facts fit beautifully into the theory that an isometrically contracting muscle possesses mechanical potential energy like a stretched spring and does work by this means. But it is equally true that the known facts do not altogether preclude another hypothesis according to which the energy needed for muscular work is developed during the actual period of shortening.
In conclusion it appears that during the reversal of a limb, the muscles are continuously innervated so that tension is maintained, redevelopment of tension for the back stroke is avoided and the energy equivalent of a certain amount of oxygen is saved. Such a saving of oxygen does not mean however a saving of mechanical energy. The kinetic energy observed in the return stroke may nevertheless have to be redeveloped de novo in spite of the fact that the necessary tension is still there. As a guess it might be said that the storage of energy could not be over 25 per cent of the kinetic energy of the limb before reversal.

Work of deceleration. The work of deceleration is work necessary to stop a moving limb; it is tension exerted while a muscle is being stretched, or negative work. Measurements have been made in man of the extra oxygen consumption involved in such positive and negative work. Chauveau (1901) found for example that negative work involved 52 per cent as much energy as positive work. Zuntz gave a figure of 40 to 45 per cent. Cathcart (1922) has reported a considerably higher figure of 71 per cent even after allowing for the energy used in performing movements without a load. Positive work is done in spite of viscosity and negative work with the aid of viscosity. If the external work is $A$ and the viscosity is $x$, and if the oxygen consumption is assumed proportional to the work, then, using Cathcart’s figure, $A + x/A - x = 100/71$ and $x = 1/6$ of $A$. If Chauveau’s figure (52 per cent) is used, then viscosity work is 5/16 of $A$. Similarly Zuntz’s figure (40 per cent) gives 3/7 of $A$. This estimate is of somewhat doubtful value partly because we never know exactly what the antagonistic muscles are doing in movements in man and partly because we have no proper assurance that the measurements were made either during a steady state or so as to cover completely the period of recovery. Moreover, these figures do not take into account the fact that a muscle loses tension at less than the isometric rate while being stretched and at more than the isometric rate while shortening, so that the necessary rate of tension redevelopment or the heat production is less during stretching than during shortening (Fenn, 1923). Correction for this difference would make the work of viscosity less than 3/7 of $A$, so that this is probably an upper estimate at the speed employed in the experiments of Zuntz. Also the work against viscosity will vary much with the speed of movement and this factor has not been controlled and is at a maximum in the rapid movements of sprinting. For this last reason it seems safest to choose the lowest of these estimates, (i.e., 40 per cent) in allowing for the work of deceleration of the limbs. Hence if the rate of work in accelerating the limb is 1.68 horse power, the work of deceleration would be 0.4 × 1.68 or 0.67 horse power, which would seem to be a conservative estimate.

Transfer of energy across body: When the foot is in contact with the ground in running it is occupied in exerting a force $F$ backwards on the
ground for time, \( t \), so that \( Ft \) represents the momentum, \( mv \) imparted thereby to the body. Much of this goes for example not directly into the body but into the other leg which is being carried forward at a greater velocity than the body. When this leg reaches the end of its forward stroke its momentum must be shared with the body as a whole according to the law of the conservation of momentum. In this way momentum can be transmitted about the body from one part to another, by a series of inelastic impacts. It is pertinent to inquire therefore to what extent kinetic energy can disappear from one limb only to reappear in another and so be counted twice in estimating the horse power of sprinting. A partial answer to this question may be suggested in the following manner.

If \( I_1 \) is the momentum of inertia of the body around a transverse axis through the two hip joints and \( I_2 \) the moment of inertia of the leg around the same axis then, as the leg swings forward with an angular momentum of \( I_2 \omega_2 \), it is checked by the hamstring muscles and an angular momentum \( I_1 \omega_3 \) is imparted to the body, such that

\[
I_2 \omega_3 = (I_1 + I_2) \omega_3
\]  

(3)

both limb and body moving then together with an angular velocity \( \omega_3 \). The energy gained by the body is then \( \frac{I_1 \omega_3^2}{2} \) and that lost by the leg is \( \frac{I_2 (\omega_2^2 - \omega_3^2)}{2} \) and the fraction, \( f \), of the energy lost by the leg, which is transferred, is

\[
f = \frac{I_1 \omega_3^2}{I_2 (\omega_2^2 - \omega_3^2)}
\]  

(4)

From a small cadaver studied by Braune and Fischer (1872, table 2) the following data are obtained.

<table>
<thead>
<tr>
<th>Light</th>
<th>Weight</th>
<th>T</th>
<th>e</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body and head</td>
<td>23790</td>
<td>10.57 \times 10^4</td>
<td>30.14 cm.</td>
<td>32.2 \times 10^6</td>
</tr>
<tr>
<td>Leg</td>
<td>7840</td>
<td>4.87</td>
<td>32.74</td>
<td>13.2 \times 10^6</td>
</tr>
</tbody>
</table>

\( I \) was calculated from the formula \( I = T + Me^2 \) where \( M \) is the weight, \( T \) the moment of inertia around the center of gravity and \( e \) the distance of the center of gravity from the hip joint. Using these values of \( I_1 \) and \( I_2, \omega_3 \) may be calculated in terms of \( \omega_2 \) from equations (3). Thus \( \omega_3 = \frac{13.2}{45.4} \omega_2 \) and \( \omega_3^2 = 0.085 \omega_2^2 \). This value for \( \omega_3^2 \) can now be substituted in equation
and the fraction of energy transferred becomes equal to

\[ \frac{32.2}{13.2} \times \frac{0.085}{0.915} = \frac{1}{4.4}. \]

This would seem to be a maximum figure for several reasons. It assumes the knee completely extended and rigid. When the knee is bent \( I_2 \) may be \( 5 \times 10^4 \) whence the fraction transferred is only \( \frac{1}{8.5} \). Moreover when one leg is going forward the other leg is going back so that the body is being twisted simultaneously in opposite directions. Thus neither leg will be able to twist the body and hence no energy can be transferred. The body is thus steadied by the opposite limbs in such a way that it behaves as if its moment of inertia or mass were much larger than it is. Hence the energy transfer is far less.

In this connection it is worth pointing out that the body itself has been chosen as a reference. If we had chosen the ground instead of the body as a reference point the problem of this transfer of energy would have involved an inelastic impact between two bodies moving at different speeds, i.e., the body of mass \( m_1 \) would be moving at velocity \( v_1 \) in relation to the ground while the leg of mass \( m_2 \) would be moving, during its forward stroke, for example, with a velocity \( v_2 \). For the sake of simplicity its angular movement may be neglected. Then

\[ m_2 v_2 + m_1 v_1 = (m_1 + m_2) v_3 \]

and using values for \( m_1 \) and \( m_2 \) given above and taking \( v_1 \) and \( v_2 \) as 7 and 9 meters/sec. respectively \( v_3 \) becomes 7.5 and the fraction of energy transfer is

\[ \frac{m_1 (v_2^2 - v_1^2)}{m_2 v_3^2} = 0.87. \]

As the leg swings forward it has a relatively high velocity and hence a high kinetic energy in relation to the ground. The above figure shows that at most \((7/8)\) of this energy is not degraded to heat but is transferred to the body which is thus accelerated in stopping the leg. This change in velocity of the body is real and has been measured and its magnitude will be discussed in a later paper. Obviously when the ground is used as a point of reference the danger of counting energy twice is considerable.

The present paper deals, however, only with the kinetic energy of the limbs. To disappear from one limb and reappear in another, energy must be transferred from leg to body and again from body to leg. Utilizing similar methods in this case, it may be found that \( \frac{1}{2.4} \) of energy from a body moving with angular velocity \( v_3 \) may be transferred to a stationary leg. Thus the total energy transferred from leg to leg is only

\[ \frac{1}{4.4 \times 2.4} \]

or about \( \frac{1}{10} \).
There remains the important question of transfer of energy from the upper to the lower leg or vice versa. In particular take the case where the thigh is moving forward and is checked while the lower leg continues to move forward. It may be thought of as "snapping" forward like a whip. Whether in this case there is any appreciable energy transfer needs no discussion for it can be answered experimentally from the data at hand. When the kinetic energy disappears from the upper leg a corresponding amount should appear in the lower leg. A study of figure 4 shows that in this runner, at least, the successive increases of kinetic energy in the lower leg cannot be derived in appreciable degree from the upper leg. Instead the kinetic energy contents of both upper and lower legs tend to increase and decrease more or less together. To test this point for all runners the kinetic energy of the whole leg was determined for each point in the running cycle by adding together the figures obtained for upper and lower legs separately. The successive increases in kinetic energy of this combined curve were then determined and added together. The resulting sum is shown in column 14 of table 2. These figures are all slightly less than the corresponding figures of column 13 which were obtained by adding together the separate increases of the upper and lower legs. On the average, however, the difference is small, 22.8 kgm. m. as compared to 25.1 kgm. m. or a 9 per cent difference. Hence at most 0 per cent of the kinetic energy could have been counted twice. This does not necessarily mean that 0 per cent was in fact transferred from upper to lower leg or vice versa. Possibly therefore the figure 1.68 horse power for the arms and legs is 9 per cent too high and the true figure is 1.53 H.P. This small reduction, however, is completely offset by the previous estimate that the movements of the shoulders if allowed for would increase the observed kinetic energy of the limbs as a whole about 10 per cent.

These considerations make it appear probable that the figure obtained for the kinetic energy changes of the limbs is a fair representation of the actual conditions. Sideways movements of the body have necessarily been neglected as well as the contortions of the face and the contractions of the body muscles, etc. Altogether it seems that the actual output of mechanical energy by the body in sprinting is as large in relation to the oxygen consumption as when the work is measured, for example, on a bicycle ergometer. It would seem as if these mechanical factors had been unduly neglected in preference to viscosity in considering the work of running.

SUMMARY

1. The problem of muscle viscosity is discussed in its relation to the physiology of sprinting in order to show that the available evidence does not preclude the possibility that the actual external work of sprinting (exclusive of work done against viscosity) is a large fraction of the total energy expended.
2. This conclusion is then verified by measurements of moving pictures of sprinters whereby the kinetic energy of the limbs could be calculated and plotted as a function of time.

3. An average sprinter is incurring an oxygen debt at the rate of 13 horse power while he is turning out mechanical work at the rate of 2.95 horse power or with an efficiency of 22.7 per cent. This includes work against gravity (0.1 horse power), changes in velocity (0.5 horse power), acceleration of the limbs (1.68 horse power), and deceleration of the limbs (0.67 horse power). It excludes contractions of facial and body muscles, sideways movements of the body and work against viscosity or internal friction.

4. The discussion is concerned chiefly with the possibility of storage of mechanical energy in the muscles and tendons and the transfer of momentum and energy from one part of the body to another. It is concluded that these complications do not seriously interfere with the accuracy of the figures obtained for the acceleration and deceleration of the limbs.

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