THE HARMONIC ANALYSIS OF INTRAVENTRICULAR PRESSURE CURVES

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Received for publication June 30, 1928

Investigators who have devised practical forms of manometers recording by optical registration have experienced considerable difficulty in selecting the constants so that an adequate natural frequency and a sufficient degree of sensitivity exist at the same time. Frank's equation \( T = \frac{2\pi \sqrt{M'}}{E'} \) makes it clear why this is so. The natural period, \( T \), is decreased by reducing the effective mass, \( M' \), or by increasing the volume elasticity coefficient, \( E' \). There are practical limits, however, beyond which the effective mass cannot be reduced and yet retain a practical form of instrument. There is likewise a limit beyond which \( E' \) cannot be increased without reducing the ratio between pressure changes in the cardiovascular system and amplitude of record, i.e., the sensitivity. Adequate amplitude is, however, almost as important as accurate amplitude and contour; without it, smaller oscillations or bends are often difficult to locate with precision, and fine vibrations are entirely obscured when pressure gradients are steep.

In membrane manometers, the inverse relation between natural frequency and sensitivity is easily altered by changing the thickness and tension of the rubber membrane. This is illustrated by the following data derived from tests with an optical manometer devised by the author:

In cardiodynamic researches reported at various times, the writer has taken care that manometers with all three of these ratings were employed in different experiments of the same series, and, in this way, curves recorded with different ordinate values were at his disposal. Most of the published intraventricular pressure curves were obtained with manometers having a natural frequency of about 180, and a sensitivity ratio of 3.5:10; and, as a matter of fact, they have proved the most valuable for analysis. In the study of right ventricular and pulmonary pressures, the most satisfactory curves were obtained by manometers whose sensitivity was 6:10, and whose natural frequency was about 150 per second.

It becomes desirable, however, to reinvestigate the adequacy of
manometers with such lower frequencies when employed for registration 
of intraventricular pressures and to establish definitely the experimental 
limitations to their use. It seems particularly appropriate that the 
writer should do so inasmuch as Straub (1914) has questioned the validity 
of some of his work on the lesser circulation, holding that the frequency 
of the manometers used (157 per sec.) was inadequate for such studies, 
and in later communications (1923) has stated it as his belief that natural 
frequencies of 200 and 300 per second are minimal for manometers used 
to record right and left intraventricular pressures, respectively. 

Previous methods for determining the minimal requisite natural frequency. 
The minimal frequency required of a manometer in order to reproduce 
the phasic and amplitude relations of pressure fluctuations is not a matter 
of authoritative opinion; it must be established by actual physical tests. 
The methods adopted in the past have been laborious and not entirely 
satisfactory. Briefly reviewed, the following procedures have been 
adopted:—Curves were recorded by manometers having different natural 
frequencies, i.e., ranging from the highest attainable to those of a lower 

<table>
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<tr>
<th>SENSITIVITY</th>
<th>NATURAL FREQUENCY</th>
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<tr>
<td>2:10</td>
<td>233</td>
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<tr>
<td>3.6:10</td>
<td>181</td>
</tr>
<tr>
<td>6:10</td>
<td>150</td>
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order. By employing somewhat complicated corrective procedures, 
described at various times by Mach (1863), Einthoven (1894, 1895, 1903a, 
1903b, 1906), Frank (1903b, 1905), and Garten (1902), the true contour was 
established. The lowest natural frequency at which such a manom-
eter reproduces a curve that demands no essential correction is assumed 
to be the lowest permissible frequency for the experimental conditions 
obtaining. Using this method, Frank (1905) corrected many arterial 
pressure curves recorded by manometers with different natural frequencies 
and concluded that a frequency of 104 is almost sufficient to register the 
central arterial pulse correctly. In his later development of the mirror 
manometer (1910) with a natural frequency of 180 and of his optical 
spring manometer with a natural frequency of 300, he believes he has 
devised instruments which far exceed all requisites of pressure registration. 

Whether or not manometers used to record intraventricular pressures 
should have a higher or lower minimal frequency than those designed for 
recording arterial pressures may be disputed on a priori grounds, but no 
final decision can be rendered in this manner. It may be argued with 
Straub (1923) that since the gradient of the intraventricular pressure
curve is steeper and the amplitude much larger, a higher frequency is demanded. But it may also be pointed out that the arterial pressure curves contain many rapid adventitious vibrations which appear not to exist in the ventricles. Hence, it may be argued with equal validity that a manometer which is capable of reproducing all of these minor deflections with accuracy may also be trusted to reproduce the much smoother type of pressure fluctuation exhibited in the ventricular curve. At various times during the last 15 years, the writer has attempted to determine the minimal natural frequency required to record intraventricular pressure curves accurately. These efforts consisted in attempts to determine whether curves recorded by manometers having natural frequencies over 200 per second were more accurate than those obtained by instruments whose frequency was about 150 per second. The results were far from conclusive. At one time the corrective procedures suggested by Frank (1903, 1905) were applied to ventricular pressure curves recorded by manometers of essentially different frequencies. The method proved difficult in its application to the rounded ventricular pressure curves, and the results were not convincing. Attempts were also made to compare identical ventricular pressure curves recorded with manometers of different frequencies simultaneously or in rapid succession. Both procedures are subject to uncontrollable experimental errors. When more than one manometer cannula is inserted into a ventricle, the probability that one or both are blocked or pocketed during certain phases of ventricular systole is very great. Consequently, curves were obtained whose contours varied so strikingly that the differences could not easily be attributed to the manometers. The most consistent results were obtained by leaving a single manometer in position and changing only the recording capsules, but as this introduced a delay, circulatory conditions often changed somewhat and the curves were not strictly similar on this account. In only a few experiments was there reason to believe that the curves were similar, but the fact was difficult to establish because the amplitudes were so different. Until some convenient and reliable mechanical method is devised for altering the ordinate values of pressure curves without changing the abscissal values, there will be great difficulty in proving that two curves of differing amplitudes do or do not correspond in every detail.

There is, however, a physical method by means of which the question as to the minimal frequency required to reproduce intraventricular curves accurately can be settled. Theoretical and mathematical analyses of Frank (1911) and those of Broemser (1914, 1918) which supplement them have clearly proven that curves are accurately reproduced by a manometer—i.e., within the limitations of analysis or calculation—provided the natural frequency is at least 4 or 5 times greater than the shortest sig-
significant component entering into the true curve. Obviously, however, such a criterion is not particularly useful unless the shortest significant component is known. Now, the pressure curves from the cavities of the heart and from the large vessels like any periodic compound curve may be resolved by the method of harmonic analysis based on Fourier’s theorem into its simple harmonic components. If such a curve is analyzable into a sufficient number of components, the shortest significant one can be established and the minimal frequency required of a manometer can be precisely determined. Broemser (1918) attempted such an analysis of the arterial pressure curve of a rat, but the harmonic analyzer at his disposal was unfortunately limited to the reproduction of six components. He found this number inadequate for the resynthesis of a curve corresponding in form and amplitude to that of the original. The number of components required to do this could not be actually determined owing to the limitations of the analyzer, but he ventured the opinion that at least 30 to 50 appeared to be necessary.

Since Prof. Dayton Miller of Case School of Applied Science has developed both a Henrici harmonic analyzer and synthesizer extended to thirty components (1916), and graciously offered to analyze a series of records for me, it appeared desirable to determine whether the smallest significant component entering into the intraventricular pressure curve could be determined in this way.

**Apparatus and experimental procedure.** The intraventricular pressure curves used for analysis were recorded by the universal optical manometer described in preliminary form by the writer and Baker (1924). As the physical constants of this apparatus are of importance in analyses of this sort, they may be noted briefly in relation to the drawing of figure 1. Vertical tube, A, length, 8 cm.; internal diameter, 1.3 cm. Side tube B, length, 2.5 cm.; internal diameter, 1.3 cm. Tube C, length up to stop-cock, 2 cm.; diameter, 0.6 cm. Height of glass cupola D, 0.6 cm. Con-

1 It is interesting to note that a similar multiple of the lowest component entering into electrocardiograms was independently established by Fahr (1914) as the minimal required frequency of the string galvanometer.
ical passage in ball $E$, length, 1 cm.; diameters of openings, 0.9 and 0.5 cm. Cylindrical passage $F$, terminating in segment opening, length, 0.7 cm.; diameter, 0.4 cm. Conical connection with cannula $G$, length, 0.7 cm.; diameters of openings, 1.3 and 0.3 cm. Cylindrical passage through stopcock and cannula $H$, diameter, 0.3 cm.; length, 3.3 cm. for straight cannula and 4.5 cm. for curved arterial cannula. Diameter of segment capsule, 0.4 cm.

The effective masses $M'$ of the several portions calculated according to Frank's formula are as follows: $A = 6.0; B = 1.8; C = 7.9; D = 0.25$;

$E = 0.7; F = 5.5; G = 0.6; H = 4.7$ (straight cannula), and 6.3 (curved cannula). Total effective mass, 27.45 (straight cannula), 29.05 (curved cannula).

The vibration frequency of the manometer fitted with three different thicknesses of rubber, the logarithmic decrement, and damping factor, $D$, as defined by Frank (1903A) were determined experimentally by an adaptation of the method of Frank, i.e., by allowing the cannula to dip into a small chamber covered with highly stretched thin rubber, placing the entire system under static pressure, and suddenly bursting the elastic membrane by a hot point. Two records of such tests are shown in figure 2.
The periods of the double vibrations $T$ and the natural frequency ($N = 1/T$) are easily calculated by reference to the tuning fork below. The ratio by which the successive single vibrations decrease is the decrement.

### TABLE 2

<table>
<thead>
<tr>
<th>Vibration number</th>
<th>Amplitude</th>
<th>Natural logarithm</th>
<th>Logarithmic decrement</th>
<th>Vibration number</th>
<th>Amplitude</th>
<th>Natural logarithm</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>4.17439</td>
<td>0.16706</td>
<td>1</td>
<td>72</td>
<td>4.24850</td>
<td>0.13763</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>4.00733</td>
<td>0.15718</td>
<td>2</td>
<td>63</td>
<td>4.11087</td>
<td>0.14060</td>
</tr>
<tr>
<td>3</td>
<td>47</td>
<td>3.85015</td>
<td>0.13658</td>
<td>3</td>
<td>55</td>
<td>3.97027</td>
<td>0.14163</td>
</tr>
<tr>
<td>4</td>
<td>40.8</td>
<td>3.71357</td>
<td>0.15822</td>
<td>4</td>
<td>48</td>
<td>3.88888</td>
<td>0.13976</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>3.55535</td>
<td>0.13792</td>
<td>5</td>
<td>44</td>
<td>3.55533</td>
<td>0.13355</td>
</tr>
<tr>
<td>6</td>
<td>30.5</td>
<td>3.41743</td>
<td>0.151</td>
<td>6</td>
<td>37</td>
<td>0.490</td>
<td>0.1385</td>
</tr>
</tbody>
</table>

Average $D = 0.445$ of the Amplitude $\log$.

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**Fig. 3.** Six segments of records showing good proportionality of deflection upon standardization of optical manometer from 0 to 200 mm. Lower line, base line. Last segment shows at $X$ tests for absence of parallax of several manometers. Note precise vertical alignment. (About $\frac{1}{2}$ actual size.) (Expt. C-407-X1.)

and the natural logarithm is the logarithmic decrement. The damping coefficient $D$, is determined by Frank's equation

$$D = \frac{\lambda}{\sqrt{\pi^2 + \lambda^2}}$$

in which $\lambda$ represents the logarithmic decrement. The reliability of the procedure is attested by the close correspondence between the logarithmic decrement of succeeding vibrations which together with other data are given in table 2.
The sensitivity of the manometer was determined by calibration under static conditions and is expressed as a ratio of the deviations of the light beam on the photokymograph 1 meter distant to the pressure increase, both expressed in millimeters. The excellent proportionality between equal pressure increments and recorded deflections is shown in the curve of figure 3. Incidentally this shows that by selecting a good grade of rubber and stretching it tightly over the manometer capsules, the diminishing extension supposed to exist in a membrane manometer is reduced to an immeasurable degree within the pressure ranges used.

At a sensitivity ratio of 2:10, and with a vibration frequency of 233 per
second (fig. 1A), the volume-elasticity coefficient \( E' \) (calculated from formula \( T = 2\pi \sqrt{\frac{M'}{E'}} \)) is \( 58,982 \times 10^9 \), and, at a sensitivity ratio of \( 6:10 \) and a frequency of 150 per second (fig. 1B), is \( 24,304 \times 10^9 \).

Analysis of records. Six typical intraventricular pressure curves were selected for study. These curves were projected by a reflectoscope system so that they were enlarged exactly to a standard length of 400 mm., a requirement for use in the Henrici harmonic analyzer. I am indebted to Professor Dayton Miller of Case School of Applied Science for the harmonic analysis of the curves as well as for the measurement of the amplitudes, phase angles, etc., of successive components.

Results. The data of table 3 give a general survey of the results. It is apparent at a glance that in every case the amplitudes of components at or near the tenth become so small that they barely exceed the width of a line. When it is recalled that the redrawn curves used for analysis were enlarged approximately 5 times, we see that their amplitude is a matter of hundredths of a millimeter in the original curve. We may at once draw the probable conclusion that the smallest significant component lies somewhere between the 8th and 12th in the average curve. If this be true it should be possible to resynthesize the original curve by using such a number of components. This was done in all six experiments with the result that remarkably close correspondence with the original curves was obtained by using 10, 9, 4, 10, 11 and 9 components respectively. The knowledge that a far smaller number of significant components enters into the ventricular pressure curve than was anticipated may be of service to others who may be limited to the use of analyzers for a smaller number of components. Attempts were also made to resynthesize curves utilizing a greater number of components. Such curves, to our surprise, deviated more from the original than those synthesized from a judiciously chosen number of significant components. Whether this is the result of technical limitations in dealing with components of such small amplitude or is due to the existence of additional higher components of exceedingly small amplitude which are capable of neutralizing the unnatural deviations introduced cannot be stated. The fact remains that, in practice, curves resynthesized from approximately 10 components show the closest degree of superposition.

Two sample curves are discussed in greater detail in order that the reader may obtain a more accurate idea as to the magnitude of the deviation between the recorded and resynthesized curves. This cannot easily be done by superimposed curves, for the correspondence is so exact that differences tend to vanish when the curves are reduced to a size suitable for illustration. Consequently other devices must be resorted to. The illustration in figure 4 shows an original left ventricular pressure curve.
with a dome-shaped summit (curve 4). This was taken with a manometer having a frequency of 233 per second and factors given in table 2.

![Image of intraventricular pressure curves](http://ajplegacy.physiology.org/)

**Fig. 4.** Original intraventricular pressure curve, above; resynthesized curve, just below. Curves 1-10 first 10 components. Time, 0.02 second (expt. C-338-I).

Underneath, the first ten components are correctly redrawn as to phase and amplitude, and just beneath the original pressure record the curve resynthesized from the 10 components is drawn. These line curves were
obtained by rephotographing the original larger chart of components to such a size that the synthesized curve is a trifle smaller than the original

Fig. 5. Ventricular pressure curve, V; aortic pressure curve, A resynthesized curve, S. Curves 1–9, first 9 components. Time 0.02 second. (Expt. C-361-II.)

and hence fits beneath the recorded pressure curve as shown in figure 4. Such comparisons show: 1, that the original curve is correctly reproduced in amplitude, i.e., within limitations of measurement; and 2, that the chief
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bends in the curve, e.g., at m, n, o, p, q, r, s, t, u are reproduced by the number of components. The deviations consist of a slight difference in the gradient of the initial slow rise, m — n; a slight difference in the prominence of the humps, p and q, and a failure to reproduce the smaller variations beyond t.

Figure 5 similarly shows original aortic and ventricular pressure curves (curve 2). The latter is of the type described as having a plateau summit. Below are a series of 9 components and drawn over the original is a curve resynthesized from them. These curves were rephotographed from the large original to such a size that the resynthesized curve is a trifle larger than the original and hence fits just over the latter in the illustration. On superposition, the amplitudes were found to correspond except that the enlarged original was about 2 mm. higher. The exact correspondence of the details in the two curves is evident from the records. This precise reproduction of the original pressure curve from synthetic analysis of 9

<table>
<thead>
<tr>
<th>HEART RATE</th>
<th>DURATION OF ENTIRE CURVE</th>
<th>T. OF 10TH COMPONENT</th>
<th>N. OF 10TH COMPONENT</th>
<th>MINIMAL VIBRATION FREQUENCY (≈ 5 N)</th>
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<tbody>
<tr>
<td>140</td>
<td>0.32</td>
<td>0.032</td>
<td>31.2</td>
<td>156.0</td>
</tr>
<tr>
<td>152</td>
<td>0.291</td>
<td>0.029</td>
<td>34.4</td>
<td>172.0</td>
</tr>
<tr>
<td>171</td>
<td>0.28</td>
<td>0.028</td>
<td>35.9</td>
<td>179.5</td>
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<tr>
<td>200</td>
<td>0.26</td>
<td>0.026</td>
<td>39.0</td>
<td>195.0</td>
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<tr>
<td>229</td>
<td>0.252</td>
<td>0.025</td>
<td>40.0</td>
<td>200.0</td>
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<tr>
<td>240</td>
<td>0.240</td>
<td>0.024</td>
<td>41.6</td>
<td>208.0</td>
</tr>
<tr>
<td>260</td>
<td>0.23</td>
<td>0.023</td>
<td>43.0</td>
<td>215.0</td>
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components is of especial interest as it was recorded by a manometer having a natural frequency of only 150 per second, and constants given in table 2.

There can be no doubt that ordinarily curves of normal contour can be reconstructed from the first ten components into which the originals are resolved by harmonic analysis. If we accept the theoretical analyses of Broemser (1918) we may further conclude that a manometer having a frequency five times as great as the tenth component reproduces records that are accurate as to amplitude, phasic relations, and contour, within the limits of measurement.

The value of the 10th component obviously depends on the heart rate. In the record reproduced in figure 4, the period of the entire wave was 0.400 second and in the record of figure 5, 0.380 second, making the 10th component roughly 0.04 second. Eliminating diastolic pauses, the frequency of these 10th components thus becomes 25 per second a heart rates of 110 and 130 per minute, respectively. It is obvious that up to
these rates as accurate curves would have been recorded by a manometer with only 125 vibrations per second, and that the frequencies of the manometer employed were definitely in excess of that required in both cases. It is obvious, however, that, as the duration of the original pressure curve decreases with progressive acceleration of the heart, the value of the tenth component must also diminish and the minimal frequency requirements of a manometer must become greater. In table 4 are presented data from experiment C387 showing such calculations at heart rates from 140 to 260 per minute, the latter being about the highest speed at which the ventricle is capable of responding to supra-ventricular stimuli. The results show clearly that, according to these standards, a manometer with a natural frequency of 150 may not be used above heart rates of 140

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>N_c</th>
<th>PHASE</th>
<th>AMPLITUDE</th>
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per minute, and one with a frequency of 180 up to heart rates of about 171 per minute. These calculations assume, however, that all of these curves can be analyzed as far as 10 significant components. This appears not to be the case. Upon harmonic analysis, it is found that, as the rate increases and the curves become smoother in form and smaller in amplitude, they can usually be resynthesized by a progressively smaller number of components. The analysis of curve 3 shown in table 3 is a case in point. It corresponds to one shown as occurring at a heart rate of 260 in table 4. Analysis could not easily be extended beyond the fourth component and perfect superposition on the original resulted when a curve was resynthesized by using these four components alone. Since the fourth, the last significant component, had a period of 0.075 second and a frequency of

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The results show clearly that, according to these standards, a manometer with a natural frequency of 150 may not be used above heart rates of 140
roughly 18, a manometer with a frequency of only $5 \times 18$ or 90 per second would seem to be adequate.

While such curves may be considered to be accurate within the limits of physical measurement, it does not follow, of course, that the intraventricular pressure fluctuations are reproduced with absolute precision. The degree and nature of the distortion which still exists as to amplitude and contour can, however, be accurately determined according to the equations evolved by Broemser (1914, 1918). To do this, it is necessary first, to establish the natural frequency, $N_a$, the logarithmic decrement ($\lambda$) and damping coefficient ($D = \frac{\lambda}{\sqrt{\pi^2 + \lambda^2}}$) of the manometer.

After a pressure curve has been harmonically analyzed it is necessary to calculate for each component (1) the frequency ($N_a$), (2) the ratio $\frac{N_c}{N_a}$, conveniently designated $R_a$, (3) the amplitude in millimeters and (4) the phase angle in degrees and decimals of a degree. The spatial phase correction ($\phi$) can then be determined for each component according to the formula

$$\tan \phi = \frac{2 D R_a}{R_a^2 - 1}$$

$\phi$, so calculated, expresses the angle in decimals of a degree which must be added to the several components if the angle lies between 0 and 180°, and subtracted if it lies between 180° and 360°.

Corrections for errors in amplitude are made by multiplying the amplitude of each component by a correction factor, $Q$, calculated according to the equation:

$$Q = \frac{1}{\sqrt{(1 - R_a)^2 + 4 D^2 R_a^2}}$$

Having corrected each component in this way as to phase, angle, and amplitude, another curve can be resynthesized by using these corrected values.

In table 5, results are presented showing the corrections that would be effective for ten components in the case of the curve analyzed in figure 4, assuming that manometers with frequencies of 233 and 150 respectively had been employed. It is only necessary to scan the figures in the 7th and 10th columns showing the factor by which the amplitude must be multiplied in order to realize the exactness with which the amplitude of the original curve is reproduced. Likewise, a comparison of columns 6 and 9 clearly shows that the phasic displacement particularly of the larger components is so small that no measurable lag in the rise of the original
curve could have existed. In fact, the variations seemed so insignificant as to make it of no value to attempt resynthesis of the corrected curve. Even the maximum correction in the case of the very small 9th and 10th component is very small and the difference between the two manometers is less than 4 degrees.

SUMMARY

1. Intraventricular pressure curves were recorded by an optical manometer ranging in frequencies from 233 to 150 per second, and whose physical constants are presented. These curves were accurately enlarged, redrawn, and analyzed by means of Miller's harmonic analyzer for 30 components. Subsequently, curves were resynthesized from the components so established.

2. As the 10th component of the enlarged curve was less than 1 mm., it was considered probable that this represents, approximately, the shortest significant component. This deduction was corroborated by the fact that curves practically reduplicating the original pressure curve could be resynthesized by using the first 9 or 10 components only.

3. Following the analysis of Broemser that a manometer which has a frequency fivefold that of the shortest significant component (in these cases approximately the 10th), it was established that a manometer with a natural frequency of 125 should be capable of recording accurately the ventricular pressure curves from a dog up to heart rates of about 130 per minute, one with a frequency of 150 up to a heart rate of 140, one with a frequency of 180 practically up to the limits of speed with which the ventricle is capable of responding to supraventricular stimuli.

4. Broemser's formulae by which the phase angle and amplitude of each component may be corrected was applied, and the calculations showed that ventricular pressure curves recorded by manometers with natural frequencies of 150 to 233 per second require only insignificant corrections, i.e., such as are not capable of physical measurement.

5. The conclusion is reached that the natural frequency required to record intraventricular pressure curves accurately is much lower than has often been surmised. This fact is of practical importance as it is not necessary to sacrifice sensitivity to low natural period in the use of such recording instruments.

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