Mechanical Compliance and Resistance of the Lung-Thorax Calculated From the Flow Recorded During Passive Expiration

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Comroe, Nisell and Nims (1) have suggested that measurements upon purely passive expiration curves may be used as a rapid method for demonstrating changes in the mechanical properties of the lungs and thorax of animals and man. Essentially, their technique consists of inflating the lungs of animals or man (made apneic by deep anesthesia, curarization, or hyperventilation) to a known alveolar pressure and then permitting them to deflate passively, by elastic recoil alone, during which time measurements are made of the volume expired against time.

It is the purpose of the present study to make a more detailed study of passive expiration curves. First, a mathematical analysis has been made using the factors found by Rahn, Otis, Chadwick and Fenn (2), and Otis and Proctor (3); in this analysis, certain assumptions were required. Second, experiments were done using a model system to determine the validity of the mathematical analysis and of the assumptions. Third, experiments were done upon experimental animals and man to determine whether their passive expirations had the same characteristics as those of the model system.

This study has shown that certain lung constants can be calculated from a single passive expiration, recording expiratory flow or volume vs. time; that a valid formula can be derived to describe such expirations, and that it may be used to judge whether "passive" expirations are truly passive.

MATHEMATICAL ANALYSIS

If the lungs and thorax of an anesthetized, apneic animal be inflated by a pump, they will, on release, deflate solely by the pressure generated by the elastic recoil of these tissues. The pressure of recoil \( P \) is approximately proportional to the volume \( V \) which had been added during inflation to the functional residual capacity \((2)\).

\[
P = KV
\]  

(1)

Expiratory flow under these circumstances will be retarded by a) the frictional resistance of air moving through the airways, b) the resistance of the tissues themselves to movement, and c) by the pressure required to overcome inertia and hysteresis. During purely passive expiration the pressure of recoil, \( P \), is the total pressure available to overcome these resistances to expiration; therefore, \( P \) may also be expressed by the formula tested by Proctor and Otis (3).

\[
P = k_1\dot{V} + k_2\dot{V}^2
\]  

(2)

in which \( \dot{V} \) = flow and \( k_1 \) and \( k_2 \) are constants. \( k_1 \) includes all the resistance caused by the laminar air flow in the lungs \((4)\) and any component of tissue resistance which can be similarly described \((3)\); this will be called the viscous resistance. \( k_2\dot{V} \) includes all the resistance to turbulent air flow \((4)\) and eddies and perhaps some components of tissue resistance as well \((3)\); it will be called turbulent resistance. An assumption is made here that by confining all measurements to certain parts of the expiration curve, viz. after peak flow is attained, the factor of inertia will be so small that it can be ignored. DuBois’ calculations \((5, 6)\) indicate that this is a valid assumption and data obtained in this study confirm this and also indicate the validity of neglecting hysteresis.

Since \( P \) in equations 1 and 2 is the same, equations 1 and 2 can be combined:

\[
KV = k_1\dot{V} + k_2\dot{V}^2
\]  

(3)

There are two methods of recording passive expirations; a) by recording flow vs. time by the use of a pneumotachygraph or flow meter and b) by recording cumulative volume vs. time by use of a spirometer or body plethysmograph. Equation 3 contains both volume and flow terms. It is desirable to develop a formula which contains one or the other vs. time, but not both. Direct integration of equation 3 leads to a very complicated formula. However, by first differentiating equation 3 to obtain an equation expressed in terms of flow and its derivative, and then
integrating, it is possible to obtain a convenient function of flow and time. The solution yields:

\[
C - k_1 \log_2 (\frac{\dot{V}}{V_0}) = k_2 t
\]  

(4)

where \(C\), a constant of integration, is equal to \((k_1 \log_2 V_0 + 2k_2 \dot{V})\), \(V_0\) being the flow at time zero. \(Equation \ 4\) is suitable to apply to flowmeter records resulting from passive expirations.

There is no simple equation which can be applied directly to a spirometer tracing of cumulative volume vs. time unless one deliberately drops out either the factor for turbulent resistance or that for viscous resistance.

If one assumes that resistance to flow is largely turbulent, the term \((k_1 \log_2 V)\) can be dropped from \(Equation \ 4\) and integration of this simplified formula yields:

\[
V = \left(\sqrt{\frac{k_1}{k_2} t} - \frac{1}{2}\right)
\]

(5)

where \(V_0\) = volume at time zero.

If one assumes that resistance to flow is largely viscous, the term \((2k_2 \dot{V})\) can be dropped from \(Equation \ 4\) and integration of this simplified formula yields:

\[
V = V_0 e^{-\frac{k_1 t}{k_2}}
\]

(6)

This is the formula for an electrical analog of a condenser discharging through a resistor, which has been applied to certain respiratory problems by Radford (7).

In the following sections, \(Equation \ 4\) will be applied to analysis of flow meter records and \(Equations \ 5\) and \(6\) to spirometer tracings. \(Equations \ 5\) and \(6\) obviously will fit only under special conditions or for certain portions of the records in which the special assumptions are true.

**TESTS UPON A MODEL SYSTEM**

Before analyzing passive expiratory curves in animals, we tested a model system; the latter included the same system of valves and tubing planned for the animal experiments but a rigid container was substituted for the lungs and thorax. We followed the suggestion of Fenn (2) and employed a metal cylindrical tank of 20 gallon capacity. The orifice of the tank served as the resistance to outflow; sometimes an additional resistance (a canister of glass beads) was added (fig. 1). The pressure in the tank, relative to atmosphere was measured by a Lilly electrical capacitance manometer. The tank was ‘inflated’ to a certain pressure and then permitted to ‘expire’ by opening a solenoid valve. The ‘expired’ gas passed through the resistances \(R\) and sometimes \(R_1\) and into a carefully counterbalanced compensating spirometer of 6 liter capacity which recorded on a kymograph moving at a speed of 93 mm/sec. At the flow rates attained in these experiments, the spirometer appeared to give a faithful record of cumulative flow at each instant; similar ‘expirations’ conducted through a Lilly flowmeter and recalculated to yield cumulative volume vs. time, did not show significant differences from the spirometer tracings on the rapid kymograph.

With these data, the static pressure-volume characteristics of the tank can be obtained by graphing \(a)\) the pressure in the tank prior to opening the valve against \(b)\) the total expired gas volume measured in the spirometer after opening the valve. The value for elastance in \(column \ 1\) table 1 was obtained as the best fit for 26 such experiments; it agrees well with the elastance predicted for isothermal compression of a 20-gallon volume of gas.

However the pressure-volume curve during an ‘expiration’ may differ from the static pressure-volume curve due to changing temperatures and other effects constituting hysteresis. Therefore the ‘dynamic’ pressure-volume characteristics of the tank were examined by plotting \(a)\) the instantaneous pressure against \(b)\) the corresponding instantaneous excess volume in the tank during the course of six of these expirations. Dynamic elasticity was not significantly different from the static value; this indicates a negligible effect due to hysteresis.
TABLE 1. DATA (WITH S.E.) OBTAINED FROM MODEL EXPERIMENTS

<table>
<thead>
<tr>
<th>I. Elastance static (cm H₂O/l)</th>
<th>2. Resistance at Constant Flow</th>
<th>3. Predicted Slope (√k₁/4k₂ (√1/sec.)</th>
<th>4. Experimental Slope (√1/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank orifice only</td>
<td>12.58 ± 0.08</td>
<td>0.151 ± 0.022</td>
<td>2.44 ± 0.05</td>
</tr>
<tr>
<td>Tank orifice plus added resistor</td>
<td>12.58 ± 0.08</td>
<td>1.33 ± 0.13</td>
<td>1.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>2.76 ± 0.07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 2. Outflow of air from tank. Spirogram has been replotted to show √V vs. time.

Time period 'A': starting period; flow depends on the opening rate of the valve and on the inertia of the spirometer. Time period 'B': flow is controlled mainly by the turbulent resistance factor and by elastance. Determination of the experimental slope is made from the line fitted during this period. Time period 'C': the laminar resistance factor is of measurable and increasing importance.

Note that the volumes expired during time periods A, B, and C are in the ratios 1/10/1.

FIG. 3. Expiratory pneumotachographs of a dog to illustrate fit by equation 4.

The lines are the recorded data X = values calculated from equation 4 for the data obtained without added resistance. O = values calculated with added resistance. 'A' is the time period when flow is controlled by the opening rate of the valve and by the inertia of the chest and lung and is not used for the analysis. The remainder of the record ('B') is used. The ratio either of the time or of the volume expired in 'A' compared to 'B' is less than 1/20; i.e., over 95% of the record is described by equation 4.

The resistance to flow a) of the orifice alone and b) of the orifice plus the container of glass beads, was measured during a steady flow through the tank; this flow was obtained by connecting an air pressure line to the tank inlet (see fig. 1) with the solenoid valve open. Flow was again measured with the spirometer; the resistance therefore includes the resistance of the spirometer and attachments. Values of k₁ and k₂ were obtained for each resistance by the best fit method from a tabulation of pressure divided by flow against flow and are recorded in column 2, table 1.

These resistances may be regarded as indicating mainly a turbulent pressure gradient for flow values above 1 l/sec.; equation 5 in the form may therefore conveniently be used for flows greater than 1 l/sec. after peak flow is reached. For that portion of the data in which we may expect a linear relation between √V and t, the slope may be predicted from the independently obtained values of K and k₂ listed in columns 1 and 2 of table 1; this 'predicted slope' is listed in column 3.

Figure 2 is a record of one such 'expiration' recorded by the spirometer and replotted in the form of √V vs. t. In a passive expiration of this type, a short initial period (A of fig. 3) will be dominated by high acceleration as flow rises from its initial value of zero towards its maximum value. Correspondingly, the pressure used to overcome inertia, which must be the whole of the pressure made available by the elastic forces when flow is zero, rapidly drops off during this period. In addition, at
the beginning of period A, there is an infinite airway resistance owing to the closed shutter. This resistance drops very rapidly as the shutter opens until in 0.04 seconds, the shutter is wide open. Both of these effects end after the shutter opens until in 0.04 seconds or 5% of the expiration. During time period B, a graph of $\sqrt{V}$ vs. $t$ is linear. The slope of this portion of the out flow curve is the 'experimental slope' listed in column 4 of table 1 for comparison with the 'predicted slope' (column 3). In the terminal time period, $C$, when flows fall below 1 l/sec., neglect of the term $(k_1/K) \log_e \dot{V}$ of equation 4 in deriving equation 5 leads to an underestimate of $\sqrt{V}$ as expected.

Two additional observations indicate that errors owing to the inertia of the spirometer, and of all other parts of the system, are negligible: a) peak flow occurred within 0.04 seconds of the beginning flow. This was also the opening time of the solenoid driven shutter as measured separately by shining a light through the shutter. b) The experimental slope during time period B was the same in different experiments with differing initial velocities and accelerations. This slope is therefore independent of accelerations and, therefore, of the effects of inertia which depend on acceleration.

The quantitative results in the model system are summarized in the comparison of columns 3 and 4 of table 1. Agreement is sufficiently good to make the assumptions used in deriving equation 5 seem reasonable. But these are the same assumptions used in deriving equation 4 when the resistance includes laminar as well as turbulent terms. The model experiments are therefore considered sufficient to justify trial of equation 4 in animals.

**Experiments on Anesthetized Dogs**

Two dogs (8 and 15 kg) were used. Each was anesthetized with sodium pentobarbital 30 mg/kg i.v. and a tracheal cannula was inserted. Flow was measured with a Lilly flowmeter and recorded with a Brush pen writer; the flowmeter was calibrated against the spirometer. Apneaa was produced by giving additional pentobarbital or by hyperventilating the animal. Then (see fig. 1), with the solenoid, $V_1$, closed, the lungs were inflated to the desired volume with a hand pump and the static pressure recorded. The solenoid valve was then opened and the expired gas passed through the flowmeter, $F$, either to the air or to an attached spirometer, $S$. In some experiments, an additional resistance, $R_I$, was placed between the flowmeter and the spirometer. The static P-V relationships were determined by inflating the lungs with the hand pump to different pressures and measuring the volume expired upon release; the P-V curves were approximately linear over the range of pressures (0-20 cm H2O) used. Forty-eight expiratory flow curves were obtained in all; these were analyzed to determine whether they fit equation 4. In order to do this, it was necessary first to calculate the constants $C'$, $k_1/K$ and $k_2/K$ for each dog.1 This was done by selecting three points on each flow curve and measuring time and flow at each point. The points selected were a point near maximum flow, and two other points with flows equal to one-half and one-fourth of the flow at the first point. By use of equation 4, one obtains three simultaneous equations which are solved for the three desired constants. After determining these constants, equation 4 could be used to calculate the time corresponding to any desired flow so that a calculated curve could be constructed and compared with the actual curve. Eight additional points were usually used for the calculated curve (see fig. 3). The curve calculated from equation 4 was found to agree well with the actual curve after the point of peak flow, i.e. after the first 5% of time of the curve. However, equation 6 failed to fit the data except for those few cases in which the calculated $k_2$ was small, and equation 5 failed in all cases.

In one dog, 36 experiments were performed without added resistance and then a resistance of known magnitude ($P/\dot{V} = 0.84 + 2.5 \dot{V}$) was added in series with the tracheal cannula (see fig. 1) and six additional experiments were performed. Again the actual curve could be closely matched by a curve constructed from equation 4.

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1 It is impossible to determine all 4 constants ($C$, $k_1$, $k_2$, and $K$) from analysis of one curve. $C' = C/K$ being a constant of integration can be evaluated from the initial conditions; since $K$ is a common denominator for both $k_1$ and $k_2$, it is necessary to calculate only 2 physiological coefficients $k_1/K$ and $k_2/K$. 
The fact that equation 4, with constant coefficients, fit the data from a single expiration indicates constancy during a single expiration of the ratios, \( k_1/K \) and \( k_2/K \). However, these ratios were found to vary from one expiration to the next. This was not due to any progressive change in the condition of the animals during the experiments since there was no trend in the values with time. The turbulence factor, \( k_2/K \), had a higher mean value in those experiments with a larger initial inflation than in those with a low initial inflation, but the scatter and overlap of groups separated on this basis is large. However, \( k_3/K \), when plotted against the viscous factor, \( k_1/K \), yields a straight line. In other words, the turbulence factor increases when the viscous factor decreases (fig. 4). This direction of change indicates that it is the resistances that are changing and not the elastance, since a change in the elastance alone would have caused both resistance factors, \( k_1/K \) and \( k_2/K \), to increase proportionately. The six experiments with added resistance and the experiments on the second dog showed a similar trend (table 3 and fig. 4).

The mean value of the turbulence factor, \( k_3/K \), and the viscous factor, \( k_1/K \), were obtained for one dog without and with extra resistance. By use of this data and the known value of the resistance of the added resistor it is possible to calculate the elastance, \( K \), from the flow curves alone. This will be called the dynamic elastance in contrast with the 'static' elastance derived from the static pressure-volume curves. Once a value for \( K \) has been obtained, the corresponding resistances can be obtained from the time factors, \( k_1/K \) and \( k_2/K \). For example, the value for the viscous factor, \( k_2/K \), with added resistance is \( 0.142 \) (S.E. = \( 0.018 \)) sec\(^2/\)l. and without added resistance is \( 0.047 \) (S.E. = \( 0.002 \)) sec\(^2/\)l. The difference, \( 0.095 \) sec\(^2/\)l., should be equal to the turbulent resistance constant, \( 2.50 \) (S.E. = \( 0.11 \)) sec\(^2/\)l. of the added resistor, divided by the dog's chest-lung elastance during flow. Solving for the dynamic elastance leads to a value of \( K \) equal to 26 (S.E. = \( 5 \)) cm H\(_2\)O/\( l \). Similarly the value of \( k_1/K \) with resistance was \( 0.157 \) (S.E. = \( 0.013 \)) sec. and without added resistance was \( 0.140 \) (S.E. = \( 0.006 \)) sec., and the derived dynamic elastance 40 (S.E. = \( 41 \)) cm H\(_2\)O/\( l \). The best estimate from weighted averages (8) is 31 (S.E. = \( 6 \)) cm H\(_2\)O/\( l \). These may be compared with the value 34.1 cm H\(_2\)O/\( l \) for static elastance (table 2) obtained from the pressure-volume curve. The difference between dynamic and static elastance is not significant (\( P > .3 \)) even considering all differences in the values obtained as being differences between static and dynamic elastance, an unlikely possibility.

The mean \( k_1/K \) and \( k_2/K \) were next multiplied by the static elastance, \( K \), for each dog, and the resultant resistance coefficients, \( k_1 \), and \( k_2 \), as well as the elastance, \( K \), were tabulated (table 2) with the values reported by other investigators.

**Experiments on Humans**

It seemed of interest to determine whether a normal human expiration is a purely passive event. If so, the postulates for the anesthetized dog, should be applicable and equations 3 and 4 which had been validated in the dog, should describe the records. Expiratory pneumotachograms were obtained in three normal subjects breathing quietly and con-
### Table 2. Resistance and Elastance Compared with Previously Published Data

<table>
<thead>
<tr>
<th></th>
<th>Present author</th>
<th>Present author</th>
<th>Present author</th>
<th>Nisell (12)</th>
<th>Rohrer (4)</th>
<th>Otis (2)</th>
<th>Nims (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. of animal, kg</td>
<td>8</td>
<td>15</td>
<td>2.5</td>
<td>3†</td>
<td>70‡</td>
<td>70‡</td>
<td>70‡</td>
</tr>
<tr>
<td>k1 (cmH2O/l/sec.)</td>
<td>4.8</td>
<td>15</td>
<td>11</td>
<td>20†</td>
<td>0.8</td>
<td>2.7-4.2</td>
<td></td>
</tr>
<tr>
<td>k2 (cmH2O/l/sec.)</td>
<td>1.6</td>
<td>0.3</td>
<td>35</td>
<td>0.8</td>
<td>2.1-0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K (cmH2O/l)</td>
<td>34.1</td>
<td>21.7</td>
<td>172</td>
<td>150</td>
<td>8.5</td>
<td>15-5</td>
<td></td>
</tr>
</tbody>
</table>

The figures cited are means or ranges. The ? following many of the weights indicates an estimate. * The number following the species indicates the number of individuals tested by the author. † Nisell measured a mean resistance over the range of flow.

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The theoretical curve is a plot of equation 4 with $k_1 = 3$, $k_2 = 1.5$ (3) and $K = 8.5$ (2). The late portion of the eupneic expiration agrees approximately with the theoretical. However this agreement is not good enough to allow calculation of the values of $k_1$, $k_2$, and $K$ by fitting the data with equation 4.

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Mathematical analysis suggested and experiments on a model and on dogs indicated that the flow curves, occurring during passive expiration of the type proposed by Comroe et al. (1) for evaluating mechanical factors in respiration, could be analyzed for two constants, $k_1/K$ and $k_2/K$. The final equation (4) resulting from the mathematical analysis is based on three simple assumptions which may be expressed as: (a) a constant dynamic elastance equal to the static elastance $b$ a resistance expressable as the sum of a viscous and a turbulent term, and (a) negligible inertia.

Hysteresis, or lag in the stress-strain diagram, is usually defined in a P-V curve, in terms of the difference between the curve obtained while pressure is increasing and the curve obtained while pressure is decreasing. These two curves in any cycle form a loop which encloses the static P-V curve. In our experiments we have no measure of elastance during inspiration. But in expiration, we may regard hysteresis as the sum of all effects which make the pressure, developed at a given volume during expiration, smaller than the pressure developed at the same volume.
under static conditions. Dynamic elastance is equal to the static only if there is no stress-relaxation lag (hysteresis) in the tissues themselves or if any regional differences in pressure which may occur in the lung, equilibrate very rapidly in comparison with the rate of change of mean pressure within the lung; so that in effect, only the mean pressure is important in driving the lung-chest system. If we accept the good fit of the expiratory curves to equation 4 as validating our mathematical assumptions, equations 1 and 2, then the dynamic elastance, like the static, must be a constant independent of volume. Comparison of dynamic and static elastance now yields a measure of hysteresis during passive expiration. There was no significant difference ($P > .3$) in one dog tested by this method. However, due largely to the fact that the method involves the difference between two large experimental numbers, a significant difference ($P < .05$) could not be obtained unless the dynamic elastance were less than two-thirds the static. Therefore, we may only say that the sum of all forces tending to distort the static elastance ('overall hysteresis') was found experimentally to be less than one-third the static elastance.

Rohrer (4) proposed the formula $P = k_1 V + k_2 V^2$ to express the resistance in the airways on the basis of theoretical considerations. Since then many authors (3, 9, 10, 12) have found experimentally that this formula fits total resistance to flow including tissue resistance as well as airway resistance. It is interesting that a 'turbulent' term must be retained at all flow rates as predicted by Rohrer (4) from consideration of eddy effects at branching points in the bronchi. This work tends to confirm these authors. This method has the advantage over the interruptor method (3) in that it can examine any single outflow curve along its entire course.

Two observations which tend to confirm that inertia in the chest-lung system is negligible were made in the present experiments: a) Deceleration during the phase of diminishing flow was never so great as one-twentieth the acceleration during the phase of increasing flow (fig. 3). In these dogs, if the pressure corresponding to acceleration is proportional to acceleration, or to a sum of powers of acceleration, then that pressure on the phase of diminishing flow must always be less than one-twentieth the maximal pressure associated with inertia in the rising phase. But during the first tenth of a second before much volume has escaped, the driving pressure must stay nearly constant. Consider then the most extreme case, that in which the pressure corresponding to maximal acceleration is less than one-twentieth the driving pressure furnished by chest-lung elastance. Even then, in the falling phase, the pressure effects due to inertia will represent less than one-twentieth the driving pressure, i.e., disregarding it will lead to an error less than 5%. These observations are in accord with the results of DuBois (5, 6). b) The peak flow in the dog's expiratory record (.03-.04 sec.) coincided very nearly with the time of opening of the solenoid valve. The constancy during a single expiration of the two ratios, $k_1/K$ and $k_2/K$, seems to indicate a constant resistance and a constant elastance during the course of each expiration. However, there is a different resistance in each successive expiration, and this may well explain the fairly large scatter in the data of Otis et al. (3) obtained by the interruptor method. This is comparable to the changing resistance found by Nisell (12) in successive expirations of cats. In addition, the fact that $k_2$ increases when $k_1$ decreases may indicate that in some expirations eddy currents or true turbulence may be extended further along some of the air passages replacing laminar flow in those passages. Occlusion of some bronchi or alveoli might also exist during some expirations and not during others.

While these observations tend to prove the assumptions and applicability of equation 4 in the anesthetized, apneic, normal animal, there remain a whole group of situations in

### Table 3. Relationship of $k_1/K$ to $k_2/K$: statistical data

<table>
<thead>
<tr>
<th>Dog, resistance</th>
<th>$r$</th>
<th>$P$</th>
<th>Slope (sec/l.)</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>no added</td>
<td>-.815</td>
<td>&lt;.001</td>
<td>-0.236 ± .005</td>
<td></td>
</tr>
<tr>
<td>added resistance</td>
<td>-.863</td>
<td>&lt;.05</td>
<td>-0.36 ± 0.13</td>
<td></td>
</tr>
<tr>
<td>no added</td>
<td>-.952</td>
<td>&lt;.01</td>
<td>-0.18 ± 0.17</td>
<td></td>
</tr>
</tbody>
</table>

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While these observations tend to prove the assumptions and applicability of equation 4 in the anesthetized, apneic, normal animal, there remain a whole group of situations in
which this has not been proven. Increased inertia and gross inequality of dynamic and static elastance may occur in diseased individuals. Changing resistance during the course of expiration may be marked in the presence of bronchial abnormalities, emphysema, or asthma. Nisell (12) has already demonstrated changes in resistance and elastance during the course of a single expiration in cats under the influence of parasympathetic stimulating drugs by use of an equation similar to equation 3, though in normal cats his findings are similar to those obtained in dogs. Finally, when there is no apnea, as in our unanesthetized humans, equation 4 does not apply, possibly due to the effect of changing muscular tensions.

SUMMARY AND CONCLUSIONS

Experiments with a model system and with apneic anesthetized dogs show expiratory flow patterns which fit an equation, that states that flow \( \dot{V} \) decreases with time \( t \) according to the curve \( t = -k_1/K \log \dot{V} - (2k_2/K)\dot{V} + \text{constant} \) plus a constant of integration. This was derived from the assumptions a) that the resistance, \( P/V \), equals \([k_1 + k_2 \dot{V}]\), and b) that the elastance, \( P/\dot{V} \), equals \( K \). It was possible to obtain values of \( k_1 \), \( k_2 \), and \( K \) from the expiratory flow patterns. Inertia was found to be negligible. A method is detailed for using this type of data to evaluate hysteresis. In one dog, hysteresis was found to be less than one-third the value of static elastance. Normal unanesthetized human expirations do not fit this pattern. The implication is that they are not simply 'passive'.

REFERENCES

12. Nisell, O. To be published.